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Kinematics and Singularity Analysis of a Novel Type of 3-$\underline{\text{C}}$RR 3-DOF Translational Parallel Manipulator

Abstract

A new three-degrees-of-freedom (3-DOF) translational parallel manipulator (TPM) with linear actuators, i.e., 3-$\underline{\text{C}}$RR TPM, is first proposed. The rotation singularity analysis, the inverse kinematics, the forward kinematics, and the kinematic singularity analysis of the 3-$\underline{\text{C}}$RR TPM are then performed. The analysis shows that the proposed TPM has the following kinematic merits over previous TPMs. (1) The forward displacement analysis can be performed by solving a set of linear equations. (2) The Jacobian matrix of the TPM is constant. The inverse of the Jacobian matrix can be pre-calculated, and there is no need to calculate repeatedly the inverse of the Jacobian matrix in performing the forward displacement analysis and forward velocity analysis. (3) There is no rotation singularity. (4) There is no uncertainty singularity. (5) The TPM has a fewer number of links or joints. The geometric condition for a 3-$\underline{\text{C}}$RR TPM to be isotropic is also revealed. Two additional kinematic merits exist for the isotropic 3-$\underline{\text{C}}$RR TPM. The first is that an isotropic 3-$\underline{\text{C}}$RR TPM is isotropic in its whole workspace. The second is that no calculation is needed in order to pre-determine the inverse of the Jacobian matrix. Finally, preliminary design considerations are presented.

KEY WORDS—translational parallel manipulator, kinematics, singularity analysis, isotropic manipulator, screw theory, parallel mechanism

1. Introduction

Three-degrees-of-freedom (3-DOF) translational parallel manipulators (TPMs) have a wide range of applications, such as assembly and machining. Several types of TPM have been proposed (Clavel 1990; Hervé and Sparacino 1991; Hervé 1995; Tsai 1999a; Tsai1999b; Di Gregorio and Parenti-Castelli 1998; Zhao and Huang 2000; Jin and Yang 2001; Carricato and Parenti-Castelli 2001). A systematic approach is proposed in Hervé and Sparacino (1991) and Hervé (1995) to generate TPMs based on the displacement group. Systematic studies on the generation of 3-DOF TPMs are performed using respectively screw algebra or screw theory in Frisoli et al. (2000) and Kong and Gosselin (2001). It is revealed in Di Gregorio and Parenti-Castelli (1999) that rotation singularities exist for the 3-U$\underline{\text{P}}$U TPM. This is also true for the 3-$\underline{\text{R}}$UU and 3-$\underline{\text{P}}$UU TPMs. Here and throughout, R, P, U, C, $\underline{\text{R}}$, $\underline{\text{P}}$ and $\underline{\text{C}}$ denote respectively a revolute joint, a prismatic joint, a universal joint, a cylindrical joint, an actuated revolute joint, an actuated prismatic joint and a cylindrical joint whose translational degree of freedom is actuated.

In fact, previous works on the systematic type synthesis of TPMs (Hervé and Sparacino 1991; Frisoli et al. 2000; Kong and Gosselin 2001) deal mainly with the systematic type synthesis of translational parallel kinematic chains (TPKCs). Some important issues in obtaining TPMs, such as the selection of inputs for TPMs, are not dealt with systematically. It is pointed out in Hervé and Sparacino (1991) that for all the actuators to be located on the base, three legs should be used in a TPKC. In fact, this is only a necessary condition that a 3-DOF TPM with fixed motors should meet. It does not guarantee that any set of three actuators located on the base are valid. For example, for the 3-CRR TPKC[1] with planar base and moving platform, the three R joints on the 3R platform can be actuated (Zhao and Huang 2000). However, the three translational degrees of freedom of the C joints of

1. The 3-CRR TPKC is composed of three CRR legs which were first proposed in Hervé and Sparacino (1991). The CRR leg is a serial kinematic chain composed of one C joint and two R joints in sequence. In a CRR leg, the axes of the C and R joints are parallel. A TPM composed of two CRR legs was also proposed in Hervé and Sparacino (1991).

The International Journal of Robotics Research
Vol. 21, No. 9, September 2002, pp. 791-798,

the mechanism cannot be actuated simultaneously. A proof of this result is given later in this paper.

It is important, and it is still an open issue, to find some TPMs of potential practical use from the list of 3-DOF TPKCs (Hervé and Sparacino 1991; Frisoli et al. 2000; Kong and Gosselin 2001) by studying the optimum selection of inputs. The 3-$\underline{\text{C}}$RR TPM is one of the TPMs that we have found which has some kinematic merits over previous TPMs (Kong and Gosselin 2002). The 3-$\underline{\text{C}}$RR TPM presented here is said to be new since the condition for all the translational degrees of freedom of the C joints of the 3-CRR TPKC to be actuated is revealed for the first time.

This paper is organized as follows. The geometric description of the 3-$\underline{\text{C}}$RR TPM is first given. The rotation singularity analysis is then performed. The inverse and forward kinematics are dealt with respectively in Sections 4 and 5. In Section 6, the kinematic singularity analysis of the 3-$\underline{\text{C}}$RR TPM is investigated. The geometric condition for the 3-$\underline{\text{C}}$RR TPM to be isotropic is revealed in Section 7. In Section 8, preliminary design considerations are proposed. Finally, conclusions are drawn.

2. Description of the 3-$\underline{\text{C}}$RR TPM

The 3-$\underline{\text{C}}$RR TPM (Figure 1) is composed of a base and a moving platform connected by three CRR legs in parallel. The axes of the C and R joints within the same leg are parallel. The axes of the three C joints are not all located on or parallel to a common plane. The translational degrees of freedom of the three C joints are actuated.

3. Rotation Singularity Analysis of the 3-$\underline{\text{C}}$RR TPM

Rotation singularity (Di Gregorio and Parenti-Castelli 1999) for a TPM occurs when the moving platform of a TPM can rotate instantaneously. This concept has been generalized to the constraint singularity (Zlatanov, Bonev, and Gosselin 2001) of parallel manipulators of general architecture.

As the rotation singularity analysis of a TPM is input-independent, it is more accurate to refer to it as the rotation singularity analysis of the TPKC corresponding to the TPM. This is why the rotation singularity analysis of TPMs should be performed before the kinematic analysis and the kinematic singularity analysis of TPMs, the latter being input-dependent.

In this section, it will be proven, using screw theory, that no rotation singularity exists for the 3-$\underline{\text{C}}$RR TPM proposed in Section 2.

In screw theory, the motion and constraints of a kinematic chain are represented by screw systems, which are termed as twist systems and wrench systems respectively; see, for example, Hunt (1978), Tsai (1999a), and Kumar et al. (2000). It has been pointed out in Kong and Gosselin (2001) that the wrench system of a CRR leg is a screw system of order 2, composed of all the ∞-pitch wrenches[2] whose axes are perpendicular to the axis of the C joint. The wrench system of the moving platform, which is the union (linear combination) of those of all the legs, is therefore always a wrench system of order 3 composed of all the ∞-pitch wrenches, since the axes of the C joints are fixed and satisfy the condition given in Section 2. Thus, the moving platform cannot rotate at any instant. That is to say, there is no rotation singularity for the 3-$\underline{\text{C}}$RR TPM.

One of the reviewers of this paper pointed out that, in addition to the 3-$\underline{\text{C}}$RR TPM, other TPMs, such as those proposed in Hervé and Sparacino (1991) and Carricato and Parenti-Castelli (2001), also have no rotation singularity. In fact, any TPM corresponding to a TPKC with a time-invariant ∞-pitch wrench system of order 3 has no rotation singularity. TPKCs with a time-invariant wrench system fall into two categories. The first is the set of TPKCs in which all the wrench systems of the legs are time-invariant. The second is the set of TPKCs in which the wrench system of at least one of the legs is time-dependent.

4. Inverse Kinematics of the 3-$\underline{\text{C}}$RR TPM

To study the kinematics of the 3-$\underline{\text{C}}$RR TPM, two coordinate systems, $P - X_P Y_P Z_P$ and $O - XYZ$, are attached to its moving platform and base, respectively. Let B_i denote a point on the axis of the R joint i on the moving platform, A_i is a point on the axis of the C joint i on the link connected to the base by the C joint i, A_{i0} is the point on the base which is coincident with the initial position of A_i, and $\mathbf{s}_i$ is the unit vector parallel to the axes of the C joint and the R joints in leg i. For purposes of simplification and without loss of generality, the X_P-, Y_P-, and Z_P-axes of the coordinate system $P - X_P Y_P Z_P$ are respectively parallel to the X-, Y-, and Z-axes of the coordinate system $O - XYZ$. A_i and B_i are chosen in such a way that $A_i B_i$ is perpendicular to $\mathbf{s}_i$.

Let $\mathbf{b}_i^P$ denote the position vector of B_i in the coordinate system $P - X_P Y_P Z_P$, $\mathbf{a}_i$ and $\mathbf{a}_{i0}$ are, respectively, the position vectors of A_i and A_{i0} in the coordinate system $O - XYZ$, and S_i the ith input of the 3-$\underline{\text{C}}$RR TPM.

4.1. Inverse Displacement Analysis

The inverse displacement analysis of the 3-$\underline{\text{C}}$RR TPM consists of determining the required inputs, S_i ($i = 1, 2, 3$), for a given position, $\mathbf{p}$, of the moving platform, where $\mathbf{p}$ is the vector directed from point O to point P.

As no rotation singularity exists for the 3-$\underline{\text{C}}$RR TPM, $A_i B_i$ ($i = 1, 2, 3$) is perpendicular to the axis of the C joint i at any instant, i.e.,

$$\mathbf{s}_i^{\mathrm{T}}[\mathbf{p} + \mathbf{b}_i^P - (\mathbf{a}_{i0} + S_i \mathbf{s}_i)] = 0 \qquad i = 1, 2, 3. \tag{1}$$

2. An ∞-pitch wrench is actually a couple in common usage.

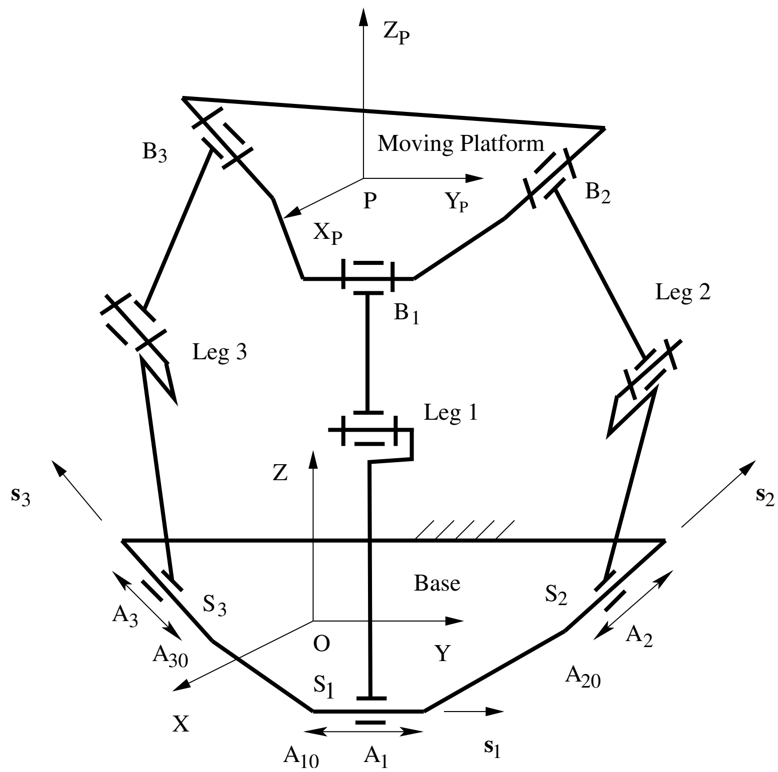


Fig. 1. Schematic description of the 3-C̲RR TPM.

Expanding eq. (1), we obtain the solution to the inverse displacement analysis

$$S_i = \mathbf{s}_i^{\mathrm{T}}(\mathbf{p} + \mathbf{b}_i^P - \mathbf{a}_{i0}) \qquad i = 1, 2, 3. \tag{2}$$

For any $\mathbf{p}$ within the workspace, the solution for the actuated joint variables, S_i, is unique. However, there usually exist two sets of real solutions for the joint variables of the passive R joints of a CRR leg. The solution for the joint variables of the R joints is omitted here for clarity as the procedure is actually the same as the inverse displacement analysis of a planar 3R serial manipulator.

4.2. Inverse Velocity Analysis

The inverse velocity analysis of the 3-C̲RR TPM consists of determining the required velocities of the actuators, $\dot{S}_i (= \mathrm{d}S_i/\mathrm{d}t)$, for a given velocity, $\mathbf{v} = (\mathrm{d}\mathbf{p}/\mathrm{d}t)$, of the moving platform in a given configuration.

Differentiating eq. (2) with respect to time, we obtain the solution to the inverse velocity analysis as

$$\dot{S}_i = \mathbf{s}_i^{\mathrm{T}}\mathbf{v} \qquad i = 1, 2, 3. \tag{3}$$

5. Forward Kinematics of the 3-C̲RR TPM

5.1. Forward Displacement Analysis

The forward displacement analysis of the 3-C̲RR TPM consists of determining the position, $\mathbf{p}$, of the moving platform for a given set of inputs, S_i ($i = 1, 2, 3$).

From eq. (1), we have

$$\mathbf{s}_i^{\mathrm{T}}\mathbf{p} = \mathbf{s}_i^{\mathrm{T}}(\mathbf{a}_{i0} + S_i\mathbf{s}_i - \mathbf{b}_i^P) \qquad i = 1, 2, 3. \tag{4}$$

Rewriting eq. (4) in matrix form, we have

$$\mathbf{J}\mathbf{p} = \begin{bmatrix} \mathbf{s}_1^{\mathrm{T}}(\mathbf{a}_{10} + S_1\mathbf{s}_1 - \mathbf{b}_1^P) \\ \mathbf{s}_2^{\mathrm{T}}(\mathbf{a}_{20} + S_2\mathbf{s}_2 - \mathbf{b}_2^P) \\ \mathbf{s}_3^{\mathrm{T}}(\mathbf{a}_{30} + S_3\mathbf{s}_3 - \mathbf{b}_3^P) \end{bmatrix} \tag{5}$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{s}_1^{\mathrm{T}} \\ \mathbf{s}_2^{\mathrm{T}} \\ \mathbf{s}_3^{\mathrm{T}} \end{bmatrix}. \tag{6}$$

Solving eq. (5), we obtain the solution to the forward displacement analysis

$$\mathbf{p} = \mathbf{J}^{-1} \begin{bmatrix} \mathbf{s}_1^{\mathrm{T}}(\mathbf{a}_{10} + S_1\mathbf{s}_1 - \mathbf{b}_1^P) \\ \mathbf{s}_2^{\mathrm{T}}(\mathbf{a}_{20} + S_2\mathbf{s}_2 - \mathbf{b}_2^P) \\ \mathbf{s}_3^{\mathrm{T}}(\mathbf{a}_{30} + S_3\mathbf{s}_3 - \mathbf{b}_3^P) \end{bmatrix}. \tag{7}$$

It should be pointed out that for a vector **p** obtained using eq. (7) with a set of valid inputs, there usually exist two sets of real solutions to the joint variables of the R joints of a CRR leg. If no real solution exists to the joint variables of the R joints of a CRR leg, then the set of inputs are invalid as the TPM cannot be assembled.

5.2. Forward Velocity Analysis

The forward velocity analysis of the 3-$\underline{\text{C}}$RR TPM consists of determining the velocity, **v**, of the moving platform for a given set of velocities of the actuators, $\dot{S}_i$ ($i = 1, 2, 3$), in a given configuration.

Rewriting eq. (3) in matrix form, we have

$$\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix} = \mathbf{J}\mathbf{v}. \tag{8}$$

Solving eq. (8), we obtain the solution to the forward velocity analysis

$$\mathbf{v} = \mathbf{J}^{-1} \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \end{bmatrix}. \tag{9}$$

5.3. Discussion on the Jacobian Matrix **J**

From eq. (6), it is clear that each row of the Jacobian matrix, **J**, is the unit vector parallel to the axis of the corresponding C joint. As the axes of all the C joints are fixed, the Jacobian matrix, **J**, is constant. As the axes of the three C joints are not all located on or parallel to a common plane (see Section 2), **J** is always non-singular and invertible.

For a given 3-$\underline{\text{C}}$RR TPM, the inverse of **J** is therefore also constant and can be pre-calculated. Thus, there is no need to calculate $\mathbf{J}^{-1}$ repeatedly in performing the forward displacement analysis and forward velocity analysis of the 3-$\underline{\text{C}}$RR TPM. This simplifies to a great extent the real-time control of the 3-$\underline{\text{C}}$RR TPM.

For a 3-$\underline{\text{C}}$RR TPM in which the translational degrees of freedom of the C joints are actuated simultaneously, eqs. (1)–(9) should be true. For the 3-CRR TPKC with planar base and moving platform (Zhao and Huang 2000), the axes of the C joints are coplanar, the Jacobian matrix, **J**, is thus always singular. As a consequence, eqs. (7) and (9) are not true. As a by-product of this section, it is proven that, for the 3-CRR TPKC with planar base and moving platform, all the translational degrees of freedom of the C joints of the mechanism cannot be actuated simultaneously.

6. Kinematic Singularity Analysis of the 3-$\underline{\text{C}}$RR TPM

6.1. Inverse Singularity Analysis

The inverse singularities for a parallel manipulator occur when the order of the twist system of any one of the legs decreases instantaneously. For the CRR leg, an inverse singularity occurs if and only if the axes of all the C and R joints are coplanar. In this case, the two solutions to the joint variable of the R joints in the CRR leg coincide with each other. This configuration corresponds to a boundary of the workspace.

6.2. Uncertainty Singularity Analysis

When an uncertainty singularity occurs for a parallel manipulator, the moving platform can undergo infinitesimal or finite motion when the inputs are locked. It is proven below that no uncertainty singularity exists for the 3-$\underline{\text{C}}$RR TPM.

From Section 3, it is known that no rotation singularity exists for the 3-$\underline{\text{C}}$RR TPM. Thus, eq. (8) is always satisfied. Uncertainty singularities for the 3-$\underline{\text{C}}$RR TPM occur if and only if **J** is singular.

From Section 5.3, it is known that the Jacobian matrix, **J**, is a non-singular constant matrix. Thus, no uncertainty singularity exists for the 3-$\underline{\text{C}}$RR TPM.

7. Isotropic 3-$\underline{\text{C}}$RR TPM

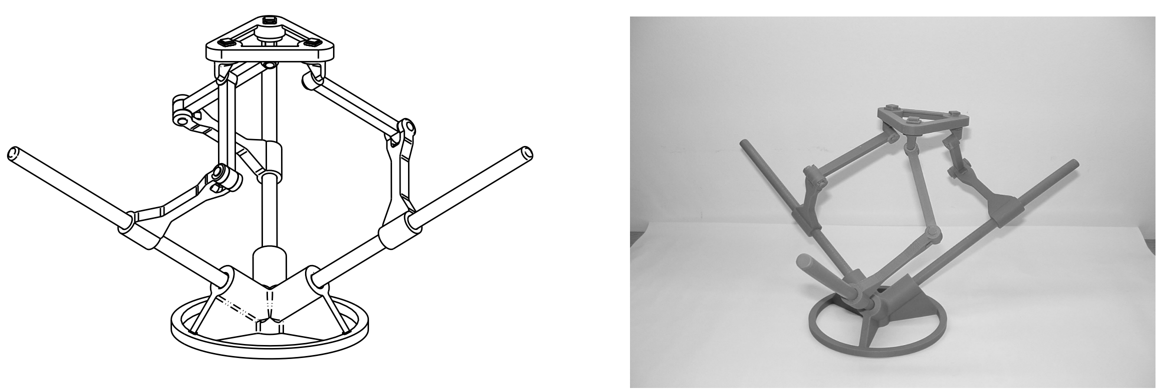
An isotropic manipulator (Angeles 1997) is a manipulator whose Jacobian matrix has a condition number equal to 1 in at least one of its configurations. In isotropic configurations, the manipulator performs very well with regard to the force and motion transmission. Isotropic manipulators proposed so far are isotropic only in a small portion of their workspace. In the following, we reveal the geometric condition which renders the 3-$\underline{\text{C}}$RR TPM isotropic and we prove that an isotropic 3-$\underline{\text{C}}$RR TPM is isotropic in its whole workspace.

As each row of the Jacobian matrix, **J**, is the unit vector parallel to the axis of one C joint (Section 5.3), it can be easily found that, when the axes of the three C joints are orthogonal, the 3-$\underline{\text{C}}$RR TPM is isotropic, i.e. the condition number of the Jacobian matrix is 1. As the Jacobian matrix, **J**, of the 3-$\underline{\text{C}}$RR TPM is constant (Section 5.3), an isotropic 3-$\underline{\text{C}}$RR TPM is isotropic in its whole workspace.

In this case, since **J** is orthogonal, we have

$$\mathbf{J}^{-1} = \mathbf{J}^{\mathrm{T}}. \tag{10}$$

Thus, no calculation is needed to obtain the inverse of the Jacobian matrix when performing the forward kinematic analysis of isotropic 3-$\underline{\text{C}}$RR TPM. Moreover, if the coordinate system $O - XYZ$ fixed on the base is defined such that vectors $\mathbf{s}_1$, $\mathbf{s}_2$, and $\mathbf{s}_3$ are, respectively, aligned with the X-, Y-, and Z-axes of $O - XYZ$, then the Jacobian matrix becomes the identity



(a) CAD model (b) Plastic model

Fig. 2. The isotropic 3-C̲RR TPM.

matrix. Hence, the inverse displacement analysis as well as the forward displacement analysis and the associated velocity problems require no computations. Each of the translation components along the X-, Y-, and Z-axes of the moving platform is directly controlled by one of the three actuators.

Using the method presented in Laliberté, Gosselin, and Côté (1999) for rapid prototyping of mechanisms based on fused deposition modeling (FDM), a plastic model of the isotropic 3-C̲RR TPM (Figure 2, see also Extension 1) has been built in the Robotics Laboratory at Laval University. The plastic model works well and the theoretical results given in previous sections have thus been verified. For a video demonstrating the motion of the model, see Extension 2.

8. Preliminary Design Considerations

8.1. The Workspace

The workspace of the 3-C̲RR TPM can be determined using a geometric approach. The geometric approach was proposed in Gosselin (1990) to determine the workspace of the Stewart platform. For a general parallel manipulator, the geometric approach can be stated as follows: the workspace of a parallel manipulator is the intersection of all its leg-spaces. Here, the leg-space is a mobility region permitted by a leg under the action of the wrench system of the moving platform. For example, a leg-space of a TPM is the mobility region permitted by a leg with the orientation of the moving platform kept constant.

For a 3-C̲RR TPM, each leg-space is bounded by two concentric cylinders whose axes are parallel to the axis of the C joint, if the limitation on joint motions and link interference are neglected. The radius of the outer cylinder is equal to the sum of the lengths of the two links in the same leg while the radius of the inner cylinder is equal to the absolute value of the difference between the lengths of the latter two links.

The maximum workspace of the isotropic 3-C̲RR TPM is a tricylinder (Figure 3) which is formed by the intersection three outer cylinders intersecting at right angles. In this case, the radius of the inner cylinder is equal to zero. (The lengths of the two links in a leg are equal.) The volume of the maximum workspace is $8(2-\sqrt{2})r^3$ (Weisstein 2001), where r is the radius of the outer cylinder.

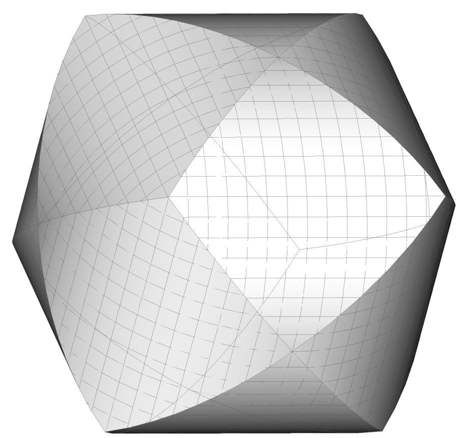


Fig. 3. The maximal workspace of the isotropic 3-C̲RR TPM.

8.2. Some Variations of the 3-C̲RR TPM

From the discussion of Section 8.1, it is clear that, for the 3-C̲RR TPM, if the stroke of the C joints is increased beyond a certain limit, the workspace of the TPM will not increase any further if the link lengths are kept constant.

Also, as pointed out in (Kong and Gosselin 2001, 2002) and by one of the reviewers of this paper, the 3-C̲RR TPM is an overconstrained mechanism. This will lead to an increased complexity of its structural design as structural analysis is necessary to determine the reaction forces.

In this subsection, some variations of the 3-C̲RR TPM are proposed in order to alleviate these two limitations.[3]

8.2.1. 3-P̲RRR TPM

To eliminate the limitation on the workspace of the 3-C̲RR TPM, we propose a variation of the 3-C̲RR TPM, namely the 3-P̲RRR TPM (Figure 4a).

It is noted that, in the 3-C̲RR TPM, each of the P joints in the C joints is used to control the position of the RRR chain along the axes of the R joints in the same leg. Without changing the function of the C joints, each C joint in the 3-C̲RR TPM can be replaced with a combination of one R joint whose axis is parallel to the axis of the C joint to be replaced, as well as one P joint whose axis is not perpendicular to the axis of the C joint to be replaced. The 3-P̲RRR TPM is thus obtained. When the axes of the P joints in the 3-P̲RRR TPM are arranged in-parallel, the workspace of the manipulator will increase linearly with an increase of the stroke of the P joints.

8.2.2. 3-C̲RRR TPM

To simplify the design and manufacturing process, a non-overconstrained TPM, namely the 3-C̲RRR TPM, which is kinematically equivalent to the 3-C̲RR TPM is proposed and is shown in Figure 4b.

The 3-C̲RRR TPM is obtained from the 3-C̲RR TPM by inserting one R joint between the moving platform and each R joint located on the moving platform. The axis of each of the new R joints is not parallel to the axes of the other R joints within the same leg. In addition to the condition on the 3-C̲RR TPM (Section 2), the insertion of the R joints should satisfy the following condition: the three vectors, each being perpendicular to all the axes of the R joints within a given leg, are not parallel to or located on one common plane. Each of the above vectors represents the direction of the ∞-pitch wrench of the corresponding CRRR leg. The additional condition guarantees that there is no rotation singularity for the 3-C̲RRR TPM. In fact, all the new R joints inserted are inactive. In other words, the joint variables of these new R joints are invariant once the TPM is assembled. Thus, the 3-C̲RRR TPM is kinematically equivalent to the 3-C̲RR TPM while the former is not overconstrained.

In practice, both non-overconstrained and overconstrained mechanisms have been widely used. To further compare the 3-C̲RR TPM with its variations, some prototypes are currently being developed in the Robotics Laboratory at Laval University.

8.2.3. Isotropic Versions of the Variations

Isotropic versions of the above variations of the 3-C̲RR TPM can also be obtained and are shown in Figure 5. The conditions for these variations of the 3-C̲RR TPM to be isotropic are that the axes of the non-inactive R joints in one leg are perpendicular to those of other legs as well as that $|\cos\alpha_1| = |\cos\alpha_2| = |\cos\alpha_3|$. Here, α_i denotes the angle between the axes of the non-inactive R joints and the axis of the P joints within leg i.

3. A systematic study of the variations of the 3-C̲RR TPM will be published later.

8.3. Comparison with other Architectures

In some of the TPMs previously proposed in the literature, some of the links are subject to tensile-compressive stresses only; see, for instance, Clavel (1990). However, in all the TPMs, there always exist some links which are subject to compound stresses, including bending.

In the architecture proposed here, all links are subject to compound stresses, which will decrease the payload/weight ratio and increase the reaction forces in the joints located on the base. However, by a careful consideration of this limitation in the design process, it is believed that very efficient prototypes can be built. For instance, reducing the friction in the joints located on the base is an important issue in the design exercise currently underway.

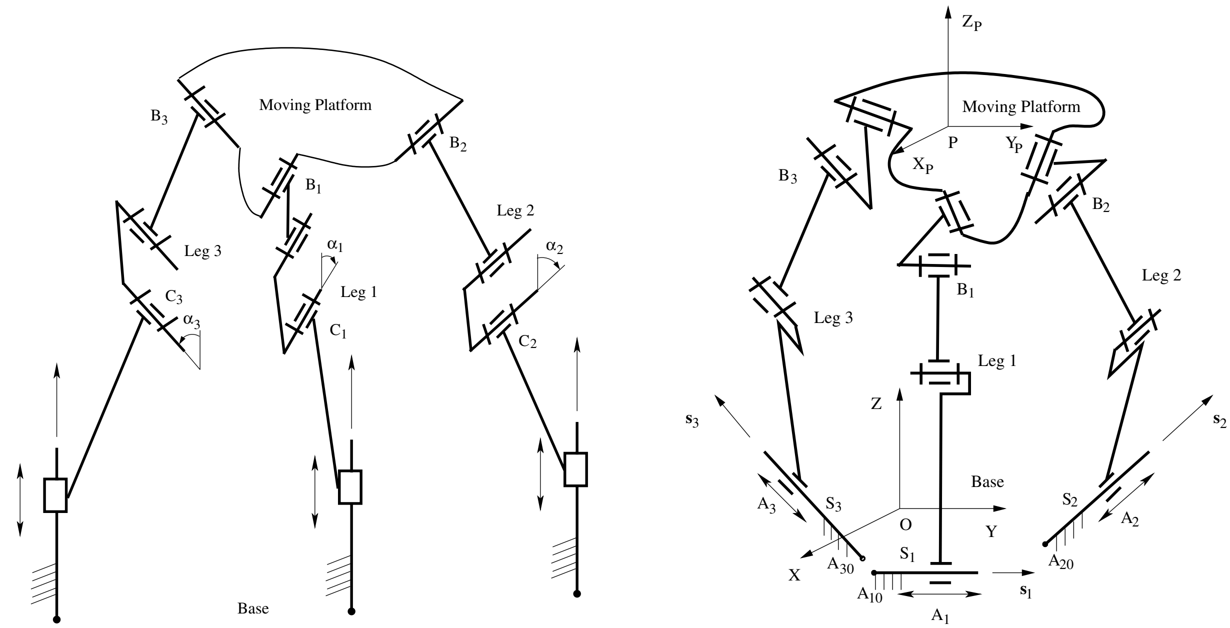
9. Conclusions

A new 3-DOF TPM with linear actuators, i.e., 3-C̲RR TPM, has been proposed in this paper. The rotation singularity analysis, the inverse kinematics, the forward kinematics, and the kinematic singularity analysis of the 3-C̲RR TPM have been performed. Preliminary practical design considerations have been discussed. It has been shown that the proposed TPM has the following kinematic merits over previous TPMs. (1) The forward displacement analysis can be performed by solving a set of linear equations. There is only one solution to the position of the moving platform for a given set of inputs, and vice versa. (2) The Jacobian matrix of the TPM is constant. The inverse of the Jacobian matrix can be pre-calculated, and there is no need to calculate repeatedly the inverse of the Jacobian matrix in performing the forward displacement analysis and forward velocity analysis. (3) No rotation singularity exists. (4) No uncertainty singularity exists. (5) The TPM has a fewer number of links or joints.

The geometric condition, which makes the 3-C̲RR TPM isotropic, has also been revealed. Two additional kinematic merits exist for the isotropic 3-C̲RR TPM. The first is that an isotropic 3-C̲RR TPM is isotropic in its whole workspace. The second is that no calculation is needed to pre-determine the inverse of the Jacobian matrix.

To the best of our knowledge, the 3-C̲RR TPM is the first parallel manipulator with a constant Jacobian matrix and the isotropic 3-C̲RR TPM is the first globally isotropic parallel manipulator.

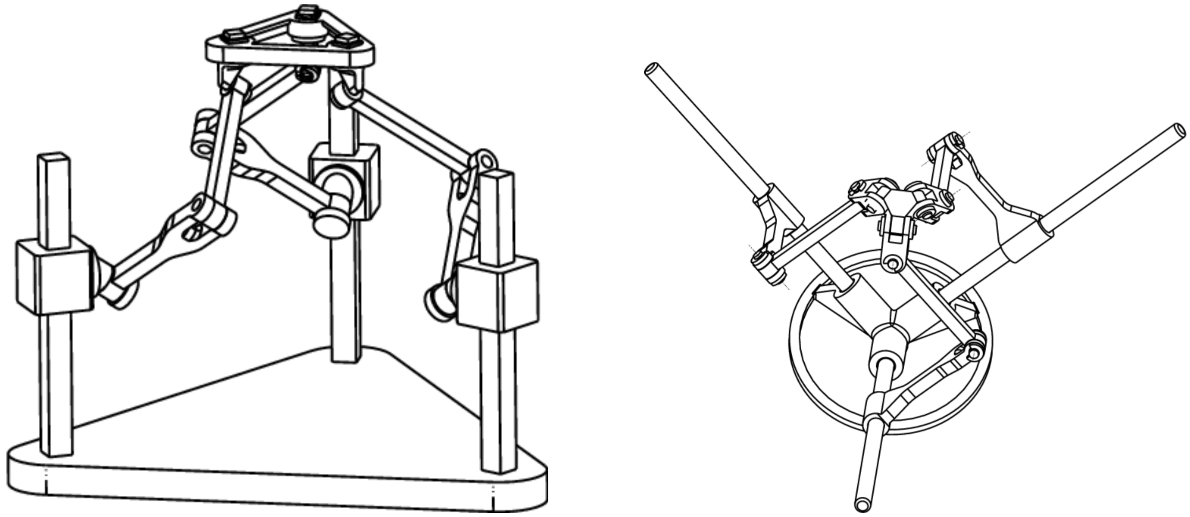
Two approaches have been adopted in the work presented in this paper, i.e., the approach based on screw theory and the method based on the differentiation of the constraint



(a) The 3-P̲RRR TPM

(b) The 3-C̲RRR TPM

Fig. 4. Some variations of the 3-C̲RR TPM.



(a) An isotropic 3-P̲RRR TPM

(b) An isotropic 3-C̲RRR TPM

Fig. 5. Some isotropic variations of the 3-C̲RR TPM.

equations. The first approach is used in the rotation singularity analysis and the inverse singularity analysis, while the second approach is used in the velocity analysis and the uncertainty singularity analysis. In this way, the above problems are solved in the most concise manner.

The results of this paper should be of great interest in the development of fast TPMs and high-performance parallel kinematic machines.

Appendix: Index to Multimedia Extensions

The multimedia extensions to this article can be found online by following the hyperlinks from www.ijrr.org.

Table 1. Index to Multimedia Extensions

Extension	Media Type	Description
1	Image	Color photograph of the plastic model of the isotropic 3-$\underline{C}$RR TPM
2	Video	Isotropic 3-$\underline{C}$RR TPM in motion

Acknowledgments

The authors would like to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC). Clément M. Gosselin holds a Canada Research Chair and would like to acknowledge the financial support of the Chair program. The financial support of Innovatech through the PVR Program is also acknowledged. Finally, the authors would like to thank Ilian Bonev for helpful discussions as well as Pierre-Luc Richard and Thierry Laliberté for building the CAD and plastic model of the isotropic 3-$\underline{C}$RR TPM.

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