ZMP Compensation by On-Line Trajectory Generation for Biped Robots

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Abstract

This paper proposes an adaptive trajectory generation strategy of using on-line ZMP information with an impedance control method. During walking, since the robot experiences various disturbances like pulling or pushing forces, the walking mechanism should have the robustness against those disturbances, and it requires an on-line adaptation ability. If sensed ZMP goes out of the preset ZMP boundary, the on-line trajectory planner generates the trajectory of the base link in the vertical direction to compensate the required moment for recovering stability. The ZMP equation and sensed ZMP information are used in this trajectory generation strategy. In this paper, an impedance controller with impedance modulation is used to control biped robots for stable walking. Computer simulation with a 3-dof environment model for which a combination of a nonlinear and linear compliant contact models is used, shows that the proposed controller performs well. Moreover, the biped robot with the proposed trajectory generator can walk even when it is pushed with a certain amount of force.

1 Introduction

During walking, biped robots experience various disturbances like pushing or pulling forces and they must be sufficiently robust against those disturbances for stable walking. The current stability of biped robots can be measured by ZMP (Zero Moment Point) information, and it can be used for the on-line moment compensation. In these context, this paper proposes an on-line trajectory generation algorithm of using ZMP equation and current ZMP information. The trajectory generator is activated if sensed ZMP goes out of a preset ZMP safety boundary and it generates the desired acceleration of base link in vertical direction to compensate a moment. Yamaguch, Takanish, and his co-workers have developed Waseda Leg series [1, 2, 3] and they used trunk motion to realize stable walking for arbitrarily planned motion trajectory, known disturbances, and ZMP trajectory. However, it is nearly impossible to find the analytical solution of trunk motion, so they used numerical iteration method in off-line manner. Only lower limb trajectories were modified in on-line manner to overcome small deviations on walking surface [3].

In this paper, an impedance control with impedance modulation strategy is used to deal with ground contacts of the swinging leg. In typical human locomotion, leg muscles are repeatedly hardened and relaxed depending on the gait phase and result in very soft contacts with the ground. Using the same idea, the parameters of the impedance control are modulated depending on the its gait phase in order to have stable contacts.

The dynamics of the biped robot is described in Section 2. The impedance controller and the impedance modulation strategy are presented in Section 3. Section 4 is devoted to the on-line trajectory generator. Section 5 describes simulations, followed by conclusions in Section 6.

2 Dynamics of Biped Robot

The biped robot used in the paper is shown in Figure 1. It has 3 degrees of freedom in each leg. Biped robots are different from the typical manipulators in that they have no fixed contact points with the ground, and the constraints between the feet and the ground change repeatedly as they walk. Actuator dynamics and friction forces in each joint are all ignored in the following equation of motion. The dynamics of

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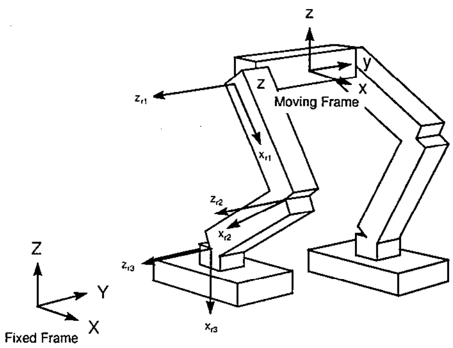

Figure 1: Coordinate frames of the biped robot

the biped robot used in the paper is described by

$$H_c\ddot{q}_c + G_c a_0 + D_c h_c + n_c = \tau_c \quad (1)$$

$$H_u\ddot{q}_u + G_u a_0 + D_u h_u + n_u = \tau_u \quad (2)$$

$$Q_c\ddot{q}_c + Q_u\ddot{q}_u + Ra_0 + P_c h_c + g = 0 \quad (3)$$

where $\ddot{q} \in \mathbb{R}^3$, $a_0 \in \mathbb{R}^6$, and $h \in \mathbb{R}^6$ are the joint acceleration, acceleration of the base link, and constrained force, respectively; $H \in \mathbb{R}^{3\times3}$, $G, D \in \mathbb{R}^{3\times6}$ are inertia matrix of the leg chain, the matrix which denotes the dynamic effects of the base link to each link chain, and a Jacobian, respectively; $n \in \mathbb{R}^3$, $Q \in \mathbb{R}^{6\times3}$, $P, R \in \mathbb{R}^{6\times6}$, and $g \in \mathbb{R}^6$ are Coriolis and centripetal term, gravitational effects, the matrix denoting the dynamic effects of the link chains to the base link, the matrix denoting the dynamic effects of the constrained force to the base link, the inertia matrix of the base link, and a term including the gravitational effects of the base link. $\tau \in \mathbb{R}^3$ is joint torque vector. Equations (1) and (2) describe the dynamics of the supporting and free legs respectively, and Eq. (3) describes the dynamics of the base link coupled with the legs. Subscripts 'c' and 'u' denotes 'constrained' and 'unconstrained', respectively. Note that the external force term in (2), h_u, is zero when the leg is in its swing phase.

3 Impedance Control of Biped Robot

Impedance controller used in this paper is identical with that in [6]. The desired impedance of the unconstrained leg at its foot is

$$M_u(\ddot{x}_{ue} - \ddot{x}_{ue,d}) + B_u(\dot{x}_{ue} - \dot{x}_{ue,d}) + K_u(x_{ue} - x_{ue,d}) = f_0 - f, \quad (4)$$

where subscript 'd' denotes the desired value, M_u, B_u, and K_u are the desired mass, damping ratio, and stiffness; and f is the resultant external force. Due to the effect of the pad stiffness and the weight which should be transferred to weight accepting leg, reference force f_0 is expressed as

$$f_0 = \begin{cases} \begin{bmatrix} 0_{2\times1} \\ K_u t_{pad} + w_t \\ 0_{3\times1} \end{bmatrix} & \text{if the pad is squeezed,} \\ 0_{6\times1} & \text{otherwise} \end{cases}$$

where w_t is the target weight. And the control input for unconstrained leg is

$$\begin{aligned} \tau_u = H_u J_{ue}^{-1}\big[&-M_u^{-1}B_u(\dot{x}_{ue} - \dot{x}_{ue,d}) \\ &- M_u^{-1}K_u(x_{ue} - x_{ue,d}) + M_u^{-1}(f_0 - f) - a_0 \\ &- \dot{J}_{ue}\dot{q}_u\big] + G_u a_0 + D_u h_u + n_u, \end{aligned} \quad (5)$$

The joint torque, τ_c for the constrained (supporting) leg can be described as

$$\begin{aligned} \tau_c = (Q_c H_c^{-1})^{-1}[\tilde{R}\{a_{0,d} - M_0^{-1}\{B_0(\dot{x}_0 - \dot{x}_{0,d}) \\ +K_0(x_0 - x_{0,d})\}\} - \tilde{P}h - \tilde{g}], \end{aligned} \quad (6)$$

where

$$\tilde{R} = Q_c H_c^{-1} G_c + Q_u H_u^{-1} G_u - R$$

$$\tilde{g} = g + Q_u H_u^{-1}(\tau_u - n_u) - Q_c H_c^{-1} n_c$$

$$\tilde{P} = \begin{bmatrix} P_c - Q_c H_c^{-1} D_c & P_u - Q_u H_u^{-1} D_u \end{bmatrix}$$

$$h = \begin{bmatrix} h_c \\ h_u \end{bmatrix}$$

which achieves the following desired impedance of base link.

$$\begin{aligned} M_0(a_0 - a_{0,d}) + B_0(\dot{x}_0 - \dot{x}_{0,d}) \\ + K_0(x_0 - x_{0,d}) = 0, \end{aligned} \quad (7)$$

where subscript '0' denotes the base link, M_0, B_0, and K_0 are the desired mass, damping ratio, and stiffness respectively.

The impedance of swinging and weight accepting leg is modified by following strategy. At the moment of the contact, the control law increases the damping ratio of the foot impedance 50 times the critical damping ratio while the desired impedance for base link is fixed for good tracking performance. The stiffness and mass components in the impedance model are selected to match the desired stiffness and the cutoff frequency of the controller [5]. This method is very simple but highly efficient to regulate the impact transition. To enhance the controller's capability in its impact regulation, the controller forces the vertical position to $z_c - t_{pad}$ and the vertical velocity to zero right after the initial contact is detected regardless of the reference trajectory of the foot.

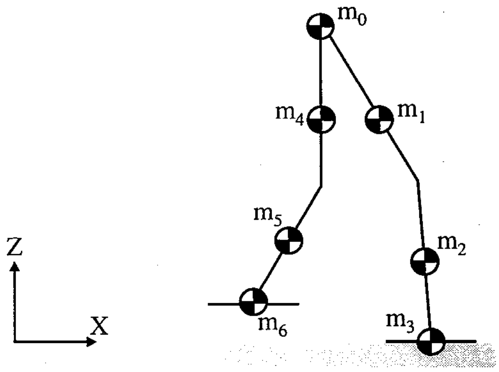

Figure 2: Model of the biped robot as a system of particles

4 On-Line ZMP Compensation

Kajita and his co-workers suggested Linear Inverted Pendulum Mode (LIPM) for trajectory generation of base link of biped robot [8]. In that paper, nonlinear ZMP equation is linearized by the assumption that the motion of base link in vertical direction is zero. Kim and Park developed Gravity-Compensated Inverted Pendulum Mode (GCIPM) which considered mass of swinging leg as well as that of base link [4]. In this paper, modified GCIPM is used for the generation of reference trajectory of base link. Because the elevation of the base link is sustained at a constant value during walking, on-line trajectory generation in vertical direction can compensate the moment for stable locomotion minimizing the effects on walking motion. In the cases of [2] and [3], the trunk motion is used for moment compensation. However, that method is applied in off-line manner and the trajectory of trunk is generated before walking. Only the lower limb trajectory is modified in on-line manner for adapting small deviations of surface in that case.

To derive on-line trajectory generation algorithm, let's start with ZMP equation. If it is assumed that the biped robot is a system of particles like Figure 2, then the ZMP in X direction can be computed by the following equation.

$$x_{zmp} = \frac{\sum_{i=0}^{6} m_i (\ddot{z}_i + g) x_i - \sum_{i=0}^{6} m_i \ddot{x}_i z_i}{\sum_{i=0}^{6} m_i (\ddot{z}_i + g)} \tag{8}$$

where g is the acceleration of gravity. Extract the terms including the acceleration of base link, $\ddot{z}_0$, from summations,

$$x_{zmp} = \frac{\sum_{i=1}^{6} m_i (\ddot{z}_i + g) x_i + m_0 (\ddot{z}_0 + g) x_0 - \sum_{i=0}^{6} m_i \ddot{x}_i z_i}{\sum_{i=1}^{6} m_i (\ddot{z}_i + g) + m_0 (\ddot{z}_0 + g)}. \tag{9}$$

If $\ddot{z}_{0,b}$ is defined as the required acceleration of the base link to fix x_{zmp} at $x_{zmp,b}$ which is the preset safety boundary of x_{zmp}, then

$$x_{zmp,b} = \frac{\sum_{i=1}^{6} m_i (\ddot{z}_i + g) x_i + m_0 (\ddot{z}_{0,b} + g) x_0 - \sum_{i=0}^{6} m_i \ddot{x}_i z_i}{\sum_{i=1}^{6} m_i (\ddot{z}_i + g) + m_0 (\ddot{z}_{0,b} + g)} \tag{10}$$

From Eq. (9) and Eq. (10), the following equation can be derived.

$$\ddot{z}_{0,b} - \ddot{z}_0 = \frac{\left\{ \begin{array}{l} (x_{zmp,b} - x_{zmp}) x_0 \sum_{i=1}^{6} m_i (\ddot{z}_i + g) \\ \quad - \sum_{i=1}^{6} m_i (\ddot{z}_i + g) x_i + \sum_{i=0}^{6} m_i \ddot{x}_i z_i \end{array} \right\}}{m_0 (x_{zmp,b} - x_0)(x_{zmp} - x_0)}. \tag{11}$$

The last equation says that in the vicinity of $x_{zmp,b} = x_0$ or $x_{zmp} = x_0$, $\ddot{z}_{0,b}$ becomes very large or infinity. Therefore, the perfect tracking of ZMP trajectory cannot be achieved by using $\ddot{z}_{0,b}$ only. Since the terms $\ddot{x}_i$'s and $\ddot{z}_i$'s are difficult to get in real implementation, Eq. (11) can be reduced to Eq. (12) only considering masses of base link, m_0, and swinging leg, m_s.

$$e_{acc} = \frac{(x_{zmp,b} - x_{zmp}) \left\{ \begin{array}{l} m_0 (\ddot{z}_0 + g)(x_0 - x_s) \\ \quad + m_s \ddot{x}_s z_0 + m_0 \ddot{x}_0 z_0 \end{array} \right\}}{m_0 (x_{zmp,b} - x_0)(x_{zmp} - x_0)}. \tag{12}$$

Although the term e_{acc} is not an exact value in magnitude, it includes sufficient direction information for ZMP compensation. Therefore, we can use it for on-line trajectory generation. When the sensed ZMP goes out of preset ZMP boundary, $x_{zmp,b}$, the trajectory generator modifies the desired acceleration of base link such as

$$\ddot{z}_{0,d} = K_{gain} e_{acc}. \tag{13}$$

If the compensator is deactivated, i.e., the sensed ZMP come into the preset boundary, then the trajectory

Table 1: The parameters of the biped robot

link	link length (m)	link mass (kg)
1	0.3	1
2	0.3	1
3	0.1	1
base	0.3	10

Table 2: The parameters of the environment model

α	0.5	k_0	2.0×10^4 N/m
k_h	1.5×10^5 N/m	b_h	1000 Ns/m

generator generates additional trajectory for returning original trajectory. 3rd order interpolation polynomial is used for additional trajectory generation.

5 Simulation

The effectiveness of the on-line trajectory generator is to be shown in computer simulations. The environment model used in this paper is identical with that of [7] except that a nonlinear spring is used. Generally, materials for pads have nonlinear stiffness which becomes very large when it is squeezed. In order to provide more realistic environmental forces, springs for vertical direction used in this environment model has nonlinear stiffness characteristics such as

$$k(p) = k_0 \left[1 + 0.1 \tan\left(\frac{\pi p}{2t_{pad}}\right)\right], \qquad (14)$$

where t_{pad} denotes the thickness of the undeformed foot pad and p denotes penetration depth. For the case of $k_0 = 2 \times 10^4 N/m$, the stiffness varies along penetration depth as shown in Figure 3. The robot parameters for the simulations are summarized in Table 1. The parameters of the environment model used in the simulations are shown in Table 2. Refer to [7] for detail descriptions of environment model. The pads underneath the feet are 1 cm thick.

Figures 4 shows ZMP trajectory and trajectory of base link when no disturbance is exerted. Although ZMP oscillates severely when a swinging leg contacts with ground, ZMP is still remains in the preset ZMP safety boundary which is set to 7cm. Since the motion in forward direction is dominant in normal walking, ZMP approaches to the upper boundary when the robot is in single support phase.

In the next simulation, a +15N disturbance pushing force in X direction is exerted at the center of base

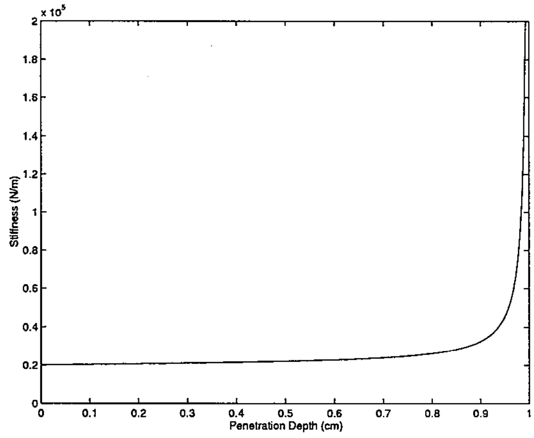

Figure 3: Stiffness vs. penetration depth

link from 1.8 second to 2.0 second. Under this disturbance, Figure 5 shows that ZMP goes out of the preset ZMP safety boundary, but it can be observed that ZMP trajectory with on-line compensation has more sufficient stability margin than that without compensation. Figure 6 shows generated desired acceleration of base link. Note that the trajectory generator makes additional reference acceleration after ZMP is in the preset ZMP boundary.

In the final simulation, the disturbance is increased to +20N. Without compensation, we can observe that ZMP in Figure 7 moves out from the preset boundary and remains at the front tip of foot. Figure 8 shows that the supporting foot becomes unstable after some bouncings. However, with compensation, Figure 9 shows that the ZMP still remains in stable region.

Those simulations show that the proposed ZMP compensation strategy realizes robust bipedal walking under short duration disturbances.

6 Conclusion

An on-line ZMP compensator with an impedance controller of using impedance modulation is proposed to control biped robots which frequently experiences various disturbances.

To investigate the performance of the proposed trajectory generator, biped robot locomotion is simulated with a 3-dof environment model with compliant contact models. The simulation results show that the biped robot with proposed trajectory generator can walk even when it is pushed with a certain amount of force.

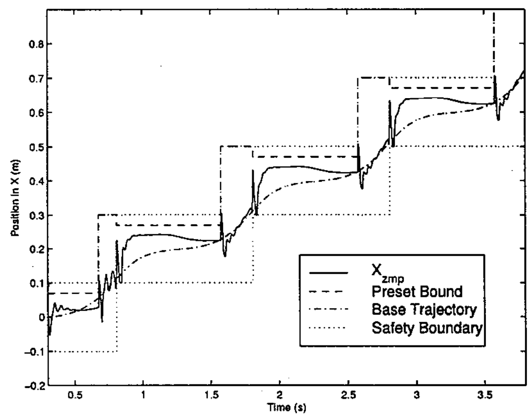

Figure 4: ZMP trajectory without disturbance

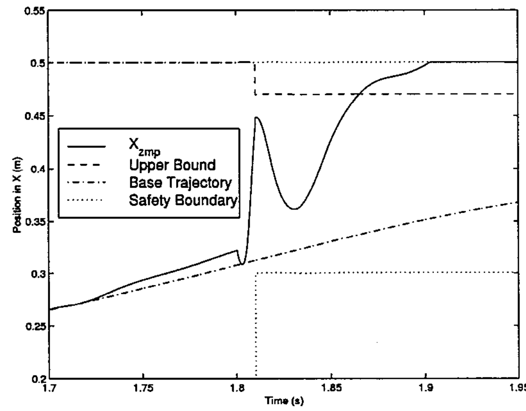

Figure 7: ZMP trajectory with +20N disturbance, without compensation

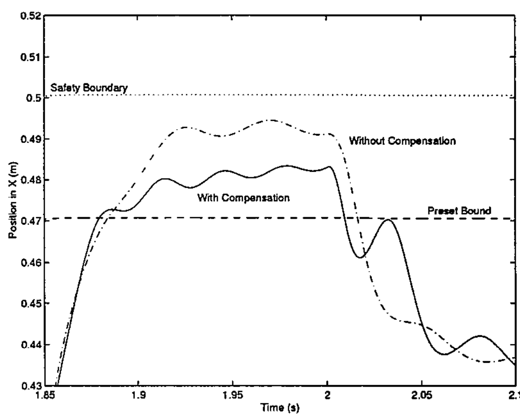

Figure 5: Closer view of ZMP trajectories comparison with +15N disturbance

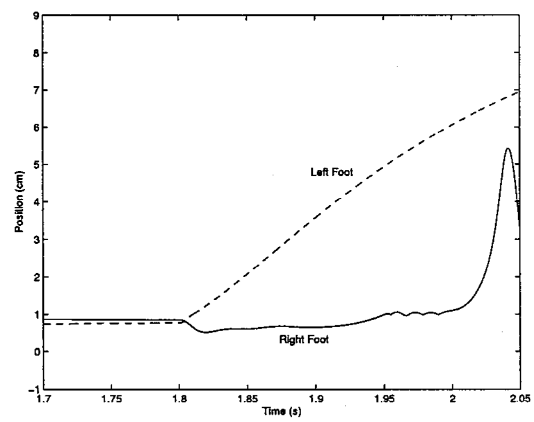

Figure 8: Feet trajectories with +20N disturbance, without compensation

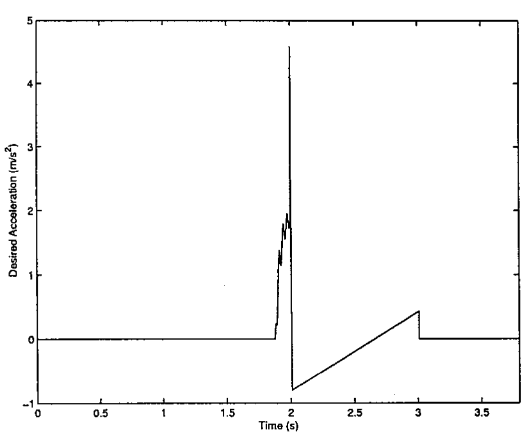

Figure 6: Desired acceleration of base link generated by on-line ZMP compensator

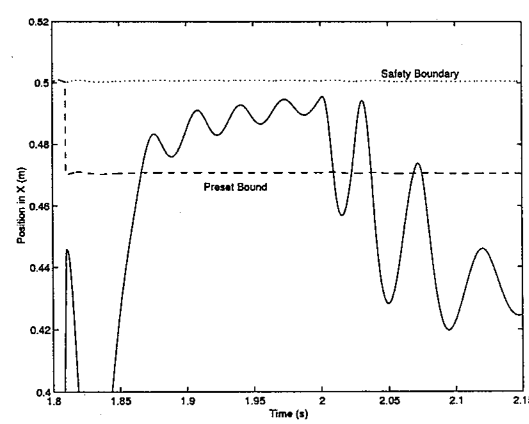

Figure 9: ZMP trajectory with +20N disturbance, with compensation

References

[1] J. Yamaguchi E. Soga, S. Inoue, and A. Takanishi, "Development of a Bipedal Humanoid Robot - Control Method of Whole Body Coorperative Dynamic Biped Walking," Proc. Int. Conf. on Robotics and Automation, pp. 368–374, 1999.

[2] J. Yamaguchi, A. Takanishi, and I. Kato, "Development of a Biped Walking Robot Compensating for Three-Axis Moment by Trunk Motion", Proc. Int. Conf. on Intelligent Robots and Systems, pp. 561–566, 1993.

[3] J. Yamaguchi and A. Takanishi, "Development of a Leg Part of a Humanoid Robot - Devlopment of a Biped Walking Robot Adapting to the Humans' Normal Living Floor," Autonomous Robots 4, pp. 369–385, 1997.

[4] J. H. Park and K. D. Kim, "Biped Robot Walking Using Gravity-Compensated Inverted Pendulum Mode and Computed Torque Control," Proc. Int. Conf. on Robotics and Automation, pp. 3528–3533, 1998.

[5] H. Kazerooni, T. B. Sheridan, and P. K. Houpt, "Robust Compliant Motion for Manipulators, Part I: The Fundamental Concepts of Compliant Motion," IEEE J. of Robotics and Automation, vol. 2, no.2, pp. 83–92, 1986.

[6] J. H. Park and Hoam Chung, "Impedance Control and Modulation for Stable Footing in Locomotion of Biped Robots," Proc. Int. Conf. on Intelligent Robots and Systems, 1999(to appear).

[7] J. H. Park and Hoam Chung, "Hybrid Control for Biped Robots Using Impedance Control and Computed-Torque Control," Proc. Int. Conf. on Robotics and Automation, pp. 1365–1370, 1999.

[8] S. Kajita, T. Yamaura, and A. Kobayashi, "Dynamic Walking Control of a Biped Robot Along a Potential Energy Conserving Orbit," IEEE Trans. on Robotics and Automation, vol. 8, no. 4, 1992.