

Biped Walking on a Low Friction Floor

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Abstract— Biped walking on a low friction floor is analyzed in this paper. For a given walking pattern, we can calculate a necessary friction coefficient which allows the robot to perform the expected motion. To reduce the maximum necessary friction coefficient, a pattern generation based on preview control theory is explained. We also describe a calculation of slip concerned ZMP, which provides a good prediction of falling caused by slips. Finally, we test a walk of 1.35 [km/h] on low friction environment using a humanoid robot HRP-2. The robot could successfully walk over the slippery area whose friction coefficient is 0.14.

I. INTRODUCTION

For a practical use of a humanoid robot, it is required to traverse on *real* environments which are not conditioned to offer flat and firm support of the feet. A robot for indoor use does not need to handle uneven or deforming ground, but it still needs to handle various level of slipperiness since a floor can become very slippery with water, oil or powders. Such conditions are dangerous not only for a robot but also for a human. For example, 3,397 Japanese died from slipping or tripping on flat ground in 2002 [1]. In the field of ergonomics and biomechanics, therefore, intensive research works have been performed to analyze slipping and slip induced falling [2]–[4]. However, it seems like there still exist controversy on the conditions which result falling [5].

Back to the field of robotics, a few works treated a biped locomotion on slippery environment. Boone and Hodgins simulated a running biped on a floor with low friction area [6]. Their original running controller could negotiate friction coefficients as low as 0.28. By introducing reflex control strategy, their robot could run over surfaces with coefficients as low as 0.025. Park and Kwon simulated a 12-DOF biped robot walking on slippery surface [7]. They designed a controller to enlarge the frictional force at slipping and it allowed their robot to traverse a surface with friction coefficient as low as 0.3.

In this paper, we examine a biped walk on slippery floor using pre-calculated walking pattern. For a given walking pattern and known friction coefficient, we want to answer the following basic questions.

Q1: Can we predict the occurrence of slip?

Q2: Can we predict falling ?

We believe answering these questions are valuable not only for robotics but also for our daily life.

The rest part of this paper is organized as follows. Section II tries to answer Q1, prediction of slipping. It also

describes a walking pattern generation which may reduce the occurrence of slip. Section III answers Q2, prediction of falling. It is shown that a simple measure based on Zero-Moment Point can well predict the occurrence of falling. Section IV describes our experiment of actual humanoid robot walking on a low friction environment. Section V summarizes the results and concludes this paper.

II. WALKING PATTERN AND FRICTION COEFFICIENT

A. Foot-Ground Interaction

Suppose a robot foot is slipping on a horizontal floor(Fig.1(a)). The foot is subjected by distributed force vectors (small black arrows) generated by microscopic interactions between the sole and the floor surface. Those force vectors can be integrated as the total floor force vector f , which can be measured by a force sensor embedded in the foot. Its tangential magnitude f_t and the vertical magnitude f_z has a relationship of

$$f_t = \mu f_z, \quad (1)$$

where μ is dynamic friction coefficient.

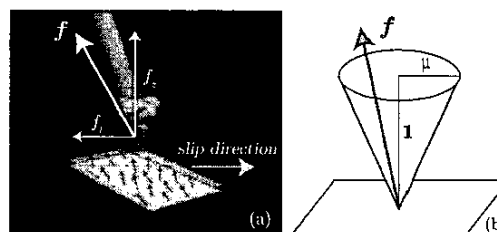


Fig. 1. Forces acting from floor to robot

When the foot is not slipping, we have

$$f_t < \mu_s f_z, \quad (2)$$

where μ_s is static friction coefficient. For simplicity, we specify $\mu_s = \mu$ in this paper. As the result, we can assume that the force vector f always lies inside or on the surface of the friction cone(Fig.1(b)).

B. Friction Requirement for a Given Walking Pattern

A humanoid robot can be modeled as a set of rigid bodies connected by joints(Fig.2). Let n be the number of joints, so the robot consists of $n + 1$ links. We define the 0-th frame on the pelvis as Σ_0 , whose position and

orientation are p_0 and R_0 with respect to the world frame Σ_w . We also define a vector of joint angles θ ($n \times 1$). A walking pattern is determined by a set of time profiles of p_0 , R_0 and θ .

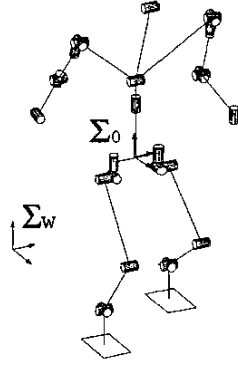


Fig. 2. A model of humanoid robot

From a given walking pattern, we can calculate the total linear momentum of the robot.

$$P = \sum_{i=0}^n m_i \dot{c}_i, \quad (3)$$

where m_i and c_i are the mass and the center of mass of the i -th link respectively.

By differentiating this linear momentum, we can obtain the vertical and tangential forces during the walking as

$$f_z = \dot{P}_z + Mg, \quad (4)$$

$$f_t = \sqrt{\dot{P}_x^2 + \dot{P}_y^2}, \quad (5)$$

where M is the total mass of the robot, g is the gravity acceleration.

If the forces f_z and f_t are not obtained from the floor, the robot will not behave as expected. Now, let us define *necessary friction coefficient* as

$$\mu_{nec} \equiv f_t / f_z. \quad (6)$$

μ_{nec} indicates the minimum friction coefficient to keep the robot on the ground without slip. The non-slip condition (2) can be rewritten as

$$\mu_{nec} < \mu. \quad (7)$$

If this inequality is satisfied for a given walking pattern, we can conclude that the robot will walk without slip.

C. Walking Pattern for a Low Friction Environment

In this section, we outline a walking pattern generation based on preview control theory [8]. With this method, we can easily modify the necessary friction coefficient of the walking pattern.

Under proper condition a walking dynamics can be approximated by

$$p_x = x - \frac{z_c}{g} \ddot{x}, \quad (8)$$

$$p_y = y - \frac{z_c}{g} \ddot{y}, \quad (9)$$

where (x, y) represent the horizontal displacement of the whole robot's center of mass (CoM), z_c is the height of the CoM and (p_x, p_y) is the Zero-Moment Point (ZMP) [9]. Fig. 3 shows a suggestive model for these equations. The CoM of the robot is represented by a running cart and the leg configuration is represented by a mass less pedestal table. A walking pattern generation is equivalent to calculate the cart motion so that it yields the prescribed ZMP.

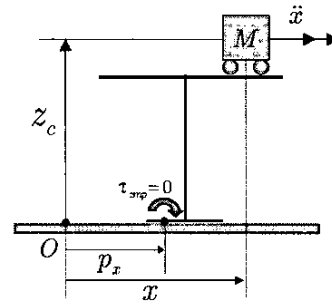


Fig. 3. A cart table model

Equation 8 can be rewritten as a standard system equation by taking the jerk ($u_x \equiv \ddot{x}$) as its input ¹.

$$\frac{d}{dt} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_x \quad (10)$$

$$p_x = \begin{bmatrix} 1 & 0 & -\frac{z_c}{g} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}.$$

The system output p_x follows the reference p_x^{ref} by applying the following *preview controller* [10].

$$u_x(k) = -G_I \sum_{i=0}^k e(i) - Gx(k) - \sum_{j=1}^{N_L} G_p(j) p_x^{ref}(k+j), \quad (11)$$

where G_I and G are the feedback gains and $G_p(j)$ is a preview gain which care about the reference ZMP up to N_L steps future. $e(i)$ is the tracking error of ZMP and $x(i)$ is the state vector,

$$e(i) \equiv p_x(i) - p_x^{ref}(i),$$

$$x(i) \equiv [x(i) \quad \dot{x}(i) \quad \ddot{x}(i)]^T.$$

¹Hereafter, we only explain the sagittal(x - z) motion but the lateral (y - z) motion can be easily obtained by the same procedure.

TABLE I
WALK PARAMETERS TO EVALUATE SLIPS AND FALLING

Height of CoM z_c :	0.814 [m]
Sagittal step length:	0.3 [m]
Lateral step width:	0.19 [m]
Step period:	0.8 [s]
Single support duration:	0.7 [s]
Double support duration:	0.1 [s]
Speed at steady walking:	0.375 [m/s]
	1.35 [km/h]
Number of steps:	4 [steps]
Total travel distance:	1.2 [m]

The gains of the preview controller are determined to minimize the following performance index.

$$J = \sum_{i=k}^{\infty} \{Q_e e(i)^2 + \Delta \mathbf{x}^T(i) \begin{bmatrix} Q_1 & 0 & 0 \\ 0 & Q_2 & 0 \\ 0 & 0 & Q_3 \end{bmatrix} \Delta \mathbf{x}(i) + R \Delta u^2(i)\}, \quad (12)$$

where Q_e, Q_1, Q_2, Q_3 and R are non-negative weights, $\Delta \mathbf{x}(i) \equiv \mathbf{x}(i) - \mathbf{x}(i-1)$ is the incremental state vector and $\Delta u(i) \equiv u(i) - u(i-1)$ is the incremental input.

Table I lists the walk parameters used to determine the reference ZMP. In all simulations and experiments of this paper, we used consistent walking patterns generated from the same reference ZMP, since the occurrence of slips and falls might be highly influenced by the walking speed, the step length and other conditions.

From the same ZMP reference, we can still generate walking patterns with different characters by adjusting the weight of (12). A walking pattern with smaller necessary friction coefficient μ_{nec} can be obtained by enlarging the weight Q_3 , because it penalizes the horizontal acceleration of CoM. We made three walking patterns for $Q_3 = 0.0, 0.5, 1.0$, and the necessary friction coefficients are shown in Fig. 4 and Table II. As expected, by enlarging Q_3 , we got a walking pattern with smaller μ_{nec} which is more suitable for lower friction environment.

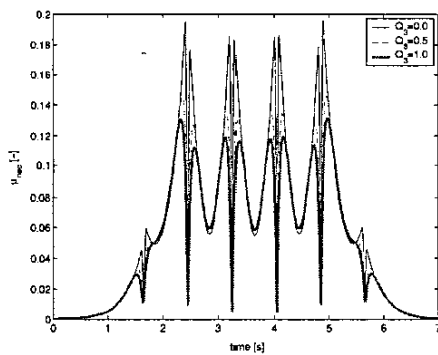


Fig. 4. Necessary friction for walking patterns

Fig. 5 shows the walking pattern of $Q_3 = 0.0$. This is a setting of our usual walking pattern generation, where the ZMP tracks the reference with good accuracy. Fig. 6

TABLE II
WALKING PATTERN AND NECESSARY FRICTION COEFFICIENTS

Walking pattern	maximum μ_{nec}
$Q_3 = 0.0$	0.196
$Q_3 = 0.5$	0.146
$Q_3 = 1.0$	0.131

Other parameters are $Q_e = 1.0, Q_1 = Q_2 = 0.0, R = 1.0 \times 10^{-6}$

is the walking pattern of $Q_3 = 1.0$, which realizes small necessary friction coefficient. As observed in the graph, the ZMP does not accurately track the reference with this setting. Nevertheless, this is also a valid walking pattern.

These patterns of CoM and reference ZMP were translated into the actual walking pattern of (p_0, R_0, θ) by using inverse kinematics.

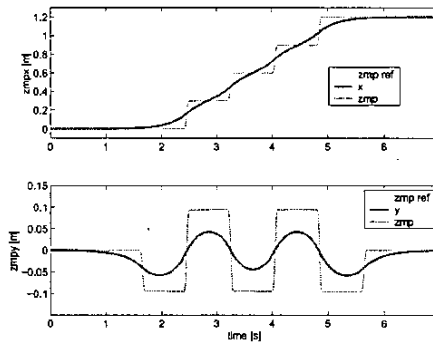


Fig. 5. Walking pattern of $Q_3 = 0$

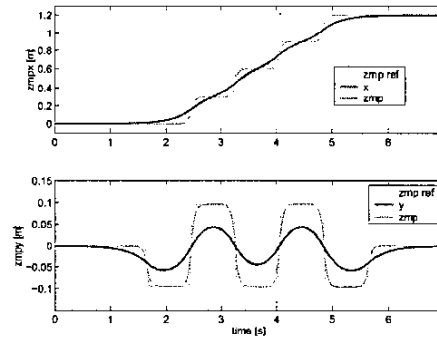


Fig. 6. Walking pattern of $Q_3 = 1.0$

D. Trajectory Error Caused by Slip

The trajectory errors caused by slips were evaluated by simulation. We used OpenHRP, which is a dynamic simulator developed in the Humanoid Robotics Project (HRP) [11]. Simulated robot model is HRP-2, a 30 DOF humanoid robot which was also developed in HRP. Fig. 7 illustrates the simulation result at $\mu = 0.08$ using the walking pattern of $Q_3 = 0$. The reference pelvis trajectory p_0^{ref} and the footholds are plotted by dotted

lines. Since slip occurred in the simulation, the simulated pelvis trajectory p_0 (bold line) and the foot placements (broken lines) did not follow the reference.

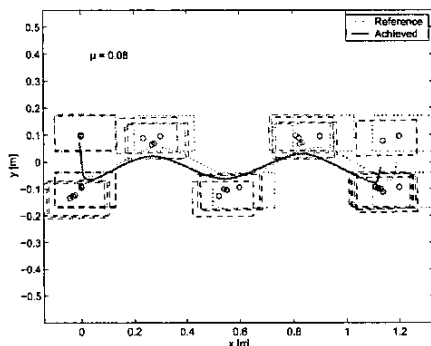


Fig. 7. Simulation result of walking on a low friction floor $\mu = 0.08$, walking pattern of $Q_3 = 0$

To evaluate the amount of slip, we defined the following index.

$$SlipIndex \equiv \int |p_0(t) - p_0^{ref}(t)| dt \quad (13)$$

Fig. 8 shows *SlipIndex* of simulated walk using three walking patterns and friction coefficients between 0.08 and 0.2.

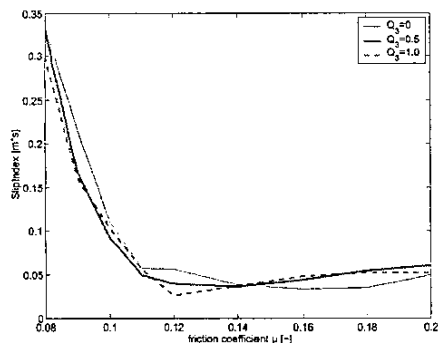


Fig. 8. Amount of slip on various friction coefficients

Although it was expected that $Q_3 = 1.0$ gives the smallest *SlipIndex* by its small μ_{nec} , it was not clear. $Q_3 = 1.0$ gave even the largest *SlipIndex* at $\mu = 0.16$ while theoretically slip will not occur at this friction coefficient. From this simulation, we could not confirm that a walking pattern having smaller μ_{nec} gives smaller amount of slip. We need further analysis on this subject as well as the check of the accuracy of OpenHRP simulator.

III. SLIP INDUCED FALLING

A. Simulated Falling

When a robot walks on a floor of very low friction, it may fall by slipping. Fig. 9 shows an example of the falling

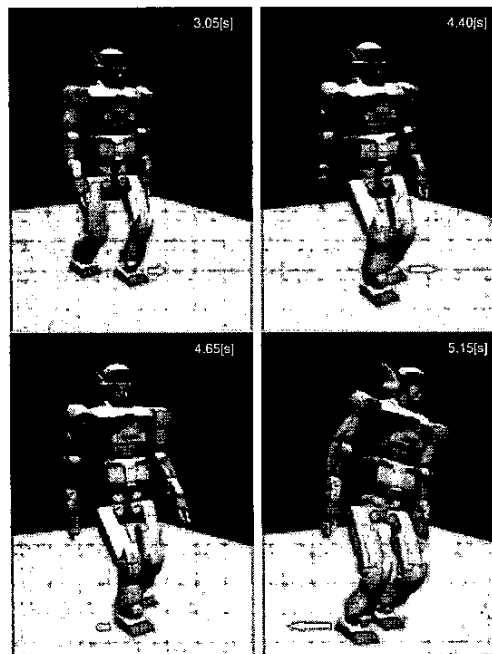


Fig. 9. Simulated slip induced falling. Walking pattern $Q_3 = 0$, $\mu = 0.05$. Arrows indicate the slip direction of the support foot.

process. We used a walking pattern of $Q_3 = 0$ and set the floor friction coefficient as 0.05.

The robot started slip toward the left at the third step (top left) and inclined because of the fast slip speed of the left foot (top right). That body inclination resulted the unexpected touchdown of the right foot, then a slip to the opposite direction started (bottom left). The slip of the right foot quickly evolved, and finally, the robot lost balance at all (bottom right). Note that the falling process was not limited to this, but many variations were observed depending on the walking pattern and the friction coefficient.

In our simulation, all walking patterns of Table II fell when $\mu \leq 0.05$. Therefore, the necessary friction coefficients μ_{nec} gives too conservative figure (more than twice) to predict the risk of slip induced falling.

B. Slip and ZMP calculation

Now, we reconsider the cart table model of Fig. 3. For simplicity, we treat the two dimensional case (sagittal motion) in this subsection.

Suppose a reference cart motion was given as $x^{ref}(t)$. According to the ZMP equation (8) the corresponding ZMP will be

$$p_x = x^{ref} - \frac{z_c}{g} \ddot{x}^{ref}.$$

However, this is valid as long as the table does not slip. Indeed, the possible acceleration of the cart is bounded by

the friction between the table and the floor.

$$\begin{aligned} f_x &= M\ddot{x} \leq \mu Mg \\ \ddot{x} &\leq \mu g \end{aligned}$$

As illustrated in Fig.10, if the cart attempts to accelerate with $\ddot{x}^{ref} > \mu g$, the table starts slipping with an acceleration of $\mu g - \ddot{x}^{ref}$ which compensates the excessive acceleration. As the result, the cart can only accelerate μg in the world frame. In this situation, the ZMP is given by the following equation.

$$p_x = x^{ref} - \mu z_c \quad (14)$$

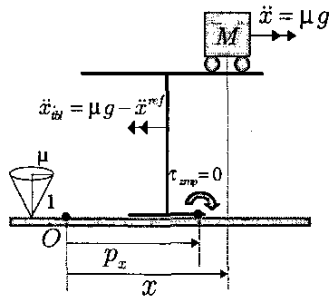


Fig. 10. Cart-table on a low friction floor: The friction coefficient between the table and the floor is μ .

C. Slip concerned ZMP

Taking into account of the above consideration, let us calculate the ZMP of a humanoid robot of Fig.2 for a given friction coefficient.

The horizontal floor reaction force f_t at slipping is

$$f_t = \mu f_z = \frac{\mu}{\mu_{nec}} \sqrt{\dot{p}_x^2 + \dot{p}_y^2}, \quad (15)$$

where μ_{nec} is the necessary friction coefficient, \mathcal{P} is the linear momentum calculated from the walking pattern.

To treat slip and non-slip condition, we introduce a parameter $\gamma \in [0, 1]$ as

$$\gamma = \begin{cases} \mu/\mu_{nec} & (\text{if } \mu_{nec} > \mu), \\ 1 & (\text{else}). \end{cases} \quad (16)$$

$0 \leq \gamma < 1$ at slipping, and $\gamma = 1$ at non-slipping. Using γ , we can calculate a slip concerned ZMP as follows.

$$p_x = c_x - \frac{\gamma \dot{p}_x c_z + \dot{\mathcal{L}}_y}{\dot{p}_z + Mg}, \quad (17)$$

$$p_y = c_y - \frac{\gamma \dot{p}_y c_z - \dot{\mathcal{L}}_x}{\dot{p}_z + Mg}, \quad (18)$$

where $[c_x, c_y, c_z]$ is the position of the total CoM and \mathcal{L} is the angular momentum around the CoM. In this paper, we refer the ZMP calculated by these equations as *slip concerned ZMP*².

²Rigorously speaking, we should say that Eq.(17) and (18) represent the *true* ZMP.

Fig. 11 illustrates this slip concerned ZMP (slip-ZMP in short) at $\mu = 0.05$ and the original ZMP. While the original ZMP trajectory (thin solid line) always runs in the middle of the support area (boundaries are shown by dashed lines), the slip-ZMP (bold line) grazes the boundaries. This indicates that the walking with low friction is less stable.

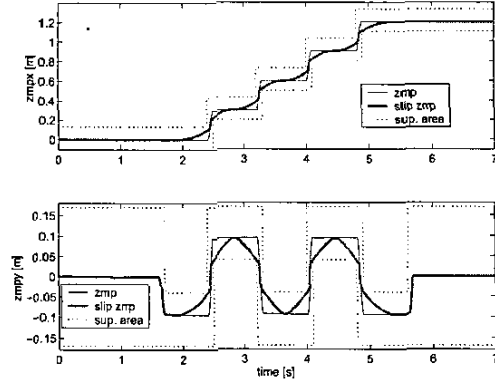


Fig. 11. ZMP, Slip concerned ZMP ($\mu = 0.05$) and support area. Walking pattern of $Q_3 = 0$

As a quantitative measure, we calculated the *StabilityIndex* which is defined by the minimum distance between the slip-ZMP and the edges of the support polygon (convex hull). *StabilityIndex* is defined to be positive when the slip-ZMP is inside of the support polygon and to be negative when the slip-ZMP is out of the polygon. Fig. 12 shows the *StabilityIndex* corresponding to Fig.11.

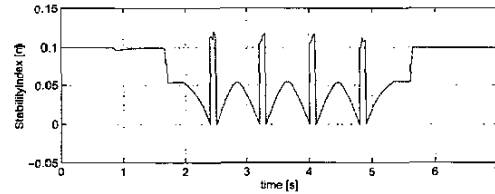


Fig. 12. *StabilityIndex* of walking pattern of $Q_3 = 0$ with ($\mu = 0.05$)

For a given walking pattern and the friction coefficient, the minimum value of *StabilityIndex* indicates the risk of falling. Fig. 13 visualizes $\min(\text{StabilityIndex})$ for the friction coefficients between 0.01 and 0.15. The plots that go less than zero at $\mu \leq 0.05$ explain the simulation results that all walking pattern fell at $\mu \leq 0.05$.

IV. WALK EXPERIMENT ON A LOW FRICTION FLOOR

In this section we describe preliminary walking experiments on low friction floors using a humanoid robot HRP-2 [12]. The soles of HRP-2 are covered by a rubber-like material, which offer the friction coefficient $\mu > 1.0$ on the lab floor. We tested two settings of low friction. The

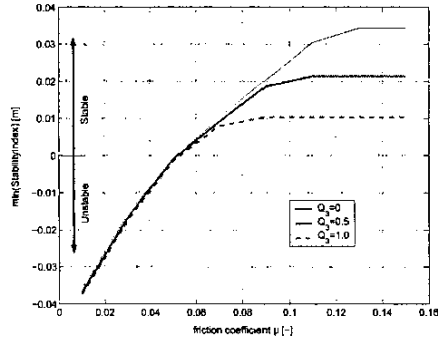


Fig. 13. $\min(\text{StabilityIndex})$ and friction coefficients

first one was a partially slippery floor with $\mu = 0.23$ and the second was a totally slippery floor with $\mu = 0.14$.

A. Walking on a floor partially $\mu = 0.23$

To obtain low friction at a specified step, we used two pieces of plastic sheet as shown in Fig. 14.

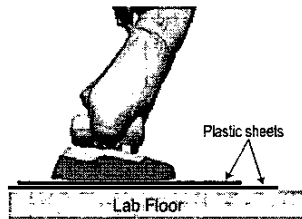


Fig. 14. Setup for low friction experiment. The friction coefficient between two plastic sheets: $\mu = 0.23$

The friction coefficient between two plastic sheets was 0.23. Putting two sheets stacked at the proper position (lower sheet was fixed to the floor), we could make the friction coefficient of the desired step to be 0.23. We tested a walking pattern of $Q_3 = 0.0$. Since $\mu = 0.23$ is large enough for the requirement of the walking pattern, we used our conventional control system without any modification. In the experiment, HRP-2 could successfully walk over the slippery area (Fig. 15).

Fig.16 shows *utilized friction coefficient* of the experiment which is defined as

$$\mu_{util} \equiv \frac{\sqrt{f_x^2 + f_y^2}}{f_z}, \quad (19)$$

where f_x , f_y and f_z are the forces measured by the foot force sensors.

The period that the robot stepped on the area of $\mu = 0.23$ is indicated by the broken line. Except spikes of high frequency, the utilized friction coefficient was well bounded by 0.23. For a comparison, Fig. 17 shows the result of the experiment without slip sheets using the same walking pattern. We can confirm that the larger friction was used for walking at the corresponding period (shown by arrows).



Fig. 15. Walk on slippery area ($\mu = 0.23$ on the black sheet, walking pattern $Q_3 = 0.0$)

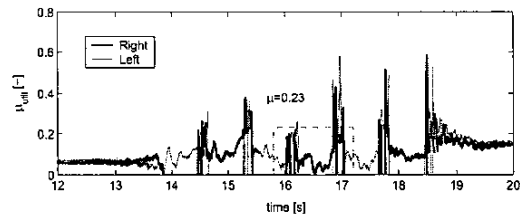


Fig. 16. Utilized friction coefficient at walking on slip area of $\mu = 0.23$

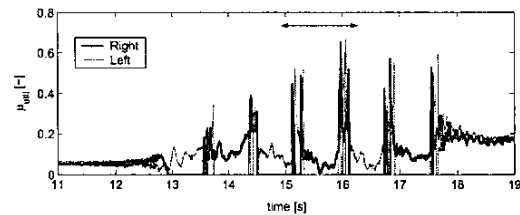


Fig. 17. Utilized friction coefficient at walking on normal floor

B. Walking on a floor $\mu = 0.14$

To obtain low- μ condition for entire walking, we attached plastic sheets on both soles of HRP-2. By putting the robot on a steel floor coated by paint, we obtained very low friction of $\mu = 0.14$. Fig.18 shows a snapshot of the experiment. We observed apparent foot slip more than 1 [cm] at every support exchange, but HRP-2 could perform reliable walking.



Fig. 18. Walk experiment on $\mu = 0.14$

To evaluate the amount of slip, we measured the travel distances for the following conditions.

- (a) With normal foot ($\mu > 1.0$)
pattern $Q_3 = 0.0, \max(\mu_{nec}) = 0.196$.
- (b) With slip foot ($\mu = 0.14$)
pattern $Q_3 = 0.0, \max(\mu_{nec}) = 0.196$.
- (c) With slip foot ($\mu = 0.14$)
pattern $Q_3 = 1.0, \max(\mu_{nec}) = 0.131$.

For each condition, ten times walking were tested. Fig. 19 shows the final positions of the right foot. The target positions are shown by rectangles with dotted line. Statistical results are listed in Table III. The robot traveled shorter than planned with ordinal friction (Fig.19 top) and traveled longer than planned with slippery condition (Fig.19 middle and bottom). It is expected that the robot will slip in condition (b) where $\mu < \max(\mu_{nec})$, and will not slip in condition (c) where $\mu > \max(\mu_{nec})$. However, the experimental results shows no obvious difference. This is unexpected, but it corresponds to the simulation result in Section II-D. It suggests us that the necessary friction coefficients does not affects the total amount of slip, but we need further investigations.

TABLE III
POSITION AND ORIENTATION ERRORS

Test	mean $\Delta x, \Delta y$ [m], $\Delta \theta$ [deg]	STD x, y [m], θ [m]
(a)	-0.033, 0.028, 0.442	0.008, 0.006, 0.609
(b)	0.052, 0.017, 1.821	0.004, 0.006, 0.524
(c)	0.051, 0.021, 1.449	0.002, 0.005, 1.011

V. CONCLUSION

In this paper, we discussed biped walking on a low friction floor. First, we defined necessary friction coefficient

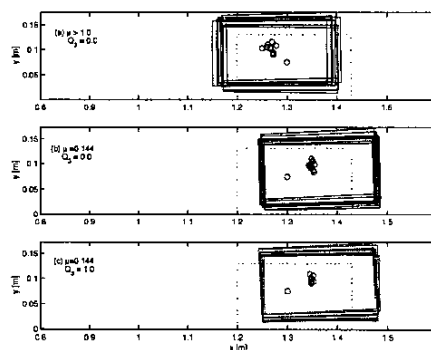


Fig. 19. Position error at 1.2[m] walk

μ_{nec} calculated from given walking pattern. Then a pattern generation method was proposed to reduce the maximum μ_{nec} . We also described a calculation of slip concerned ZMP, which provides a good prediction of falling caused by slips. Finally, we tested a walk of 1.35 [km/h] on low friction environment using a humanoid robot HRP-2. Without any modification of the control system, HRP-2 could successfully walk over the slippery area whose friction coefficient was 0.14.

Simulations suggest us that our robot can walk on a floor of ultra-low friction (ex. $\mu = 0.1$ which is said to be the friction on ice). Further analysis and experiment on such environment will be our next challenge.

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