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Kinematics Analysis Based on Screw Theory of a Humanoid Robot

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Abstract: A humanoid robot is a complex dynamic system for its idiosyncrasy. This paper aims to provide a mathematical and theoretical foundation for the design of the configuration, kinematics analysis of a novel humanoid robot. It has a simplified configuration and design for entertainment purpose. The design methods, principle and mechanism are discussed. According to the design goals of this research, there are ten degrees of freedom in the two bionic arms. Modularization, concurrent design and extension theory methods were adopted in the configuration study and screw theory was introduced into the analysis of humanoid robot kinematics. Comparisons with other methods show that: 1) only two coordinates need to be established in the kinematics analysis of humanoid robot based on screw theory; 2) the spatial manipulator Jacobian obtained by using twist and exponential product formula is succinct and legible; 3) adopting screw theory to resolve the humanoid robot arms kinematics question can avoid singularities; 4) using screw theory can solve the question of specification insufficiency.
Key words: bionics arm; screw theory; extension theory; Jacobian matrix; modularization design
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1 Introduction

Modern robots were first studied during the 1960s. Most of the conventional robots have been used in fields isolated from human society such as industry, nuclear power plants and space. With the development of robotic studies, robot applications were expanded from industrial to non-industry applications. Robots are expected to coexist with people in human society such as in homes, science and technology halls or offices. Lately, robot investigators have been paying more attention to domestic and entertainment robot applications.

Although many domestic and entertainment robots have been developed so far, some problems still exist in robot research. Generally speaking, the main problem lies in the fact that robot mechanisms are complex and extremely costly. A simplified robot and one easier to control is expected to be developed.

Mechanism is the carrier of other components in humanoid robot studies, so that the mechanism design, especially the bionic arm and robot body mechanism are the first and main concerns.

Kinematics, which includes both forward and inverse kinematics, is the basis of robot control. The comparison of forward and inverse kinematics is shown in Fig. 1.

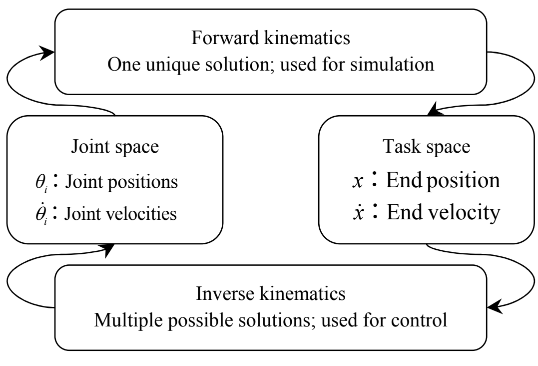

Fig. 1 Comparison of forward and inverse kinematics

Common analytical kinematic methods are the following: the coordinate transform matrix method, in which the matrix calls a Denavit-Hartenberg (D-H) matrix, the rotary transform tensor method, the plural polar vector[1] and screw theory[2].

The plural polar vector method is often used in lo-

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comotion analysis of a plane linker framework. Zhang once used this method in the literature [3]. Since the D-H parameter method has characteristics of simplicity, intuition and is easy to comprehend, it has been widely applied in kinematic analysis. Its disadvantages lie in the fact that many local reference frames are needed to set it up. Screws, twists and wrenches are usually used to describe rigid kinematic questions. Adopting screw theory to resolve the humanoid robot kinematic question can avoid singularities and predigest mechanism designs.

2 System and Mechanism Design

2.1 Design criteria

Design criteria of our proposed robot are summarized as follows.

First, the robot arm has three adjacent axes intersect at a single point. This satisfies the conditions of the existence of inverse kinematics.

Second, the robot should have a simplified framework and be inexpensive. The reason for this lies in the fact that the authors aim at making robots more common in the world.

Third, modularization, concurrent design and extension theory methods are adopted in our configuration studies. Screw theory is introduced into the analysis of humanoid robot kinematics analysis for our paper. Control, electric and sensor systems should be considered in the design process of mechanism systems. An extension theory method has been adopted to solve existing problems in robot studies, such as complex configuration, big configurations and huge feet.

The robot arm has five degrees of freedom (DOF), similar to the human arm with respect to the length and the degree of freedom. Fig. 2 illustrates the distribution of the robot DOF. DOF 1 and 2 complete turn and swing shoulder functions, DOF 3 completes the upper arm autorotation function, DOF 4 completes elbow joint swing function and DOF 5 completes the wrist swing function. The first, second and third axes are perpendicular and have a point of intersection. At the same time, the third and fourth axes are perpendicular.

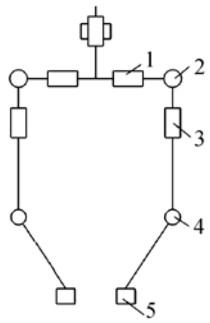

Fig. 2 Distribution of robot degrees of freedom

Finally, given the goal of our research in the mechanism design it is of fundamental importance that we first fulfill the precondition of structural intensity and full electric installation and then reduce the weight of the robot as much as possible.

2.2 Select of robot motor

The five functional DOF specification referred to above apply to each of the two robot arms. Each joint of the arm is actuated by a DC motor with a reduction gear. The DC brushless motor was selected as the robot motor; one motor controls one degree of freedom and each motor is equipped with a brushless direct current motor controller.

Brushless DC motors produce large force moments. Compared with a brush DC motor, a brushless DC motor can escape friction which a high-speed operation of rotor brush produces. Furthermore, when a brushless DC motor is working, the rotor produces three phase Hall signals which can be available to detect the operational location of the motor and its speed, while it does not need a photoelectric code wheel to carry out position control.

The robot's transmission mechanism is similar to a general mechanism, but the characteristics of the robot, determining its transmission mechanism, should be more compact in structure, smaller in volume and lighter in weight. At present, the transmission mechanism in common use in robots is a gear transmission, a turbine-worm drive mode or a timing belt transmission. Given the functional requirements of this robot, a gear transmission was selected. In addition, on the basis of the load requirements, a control actuator built-in gear reducer and a control were adopted on small load joints.

The specifications of our choice of the DC motor and other parameters are presented in Table 1.

Table 1 Specification of the arm DC motor

No.	Motor	Reachable Region	Rating electric current (A)	Motor moment (g·cm)
1	DC motor	−30°, +150°	0.744	115 200
2	DC motor	0° , +90°	0.744	115 200
3	DC motor	±45°	0.280	37 440
4	DC motor	0°, +90°	0.280	37 440
5	Steering engine	±45°	0.280	37 440

2.3 Control system

The control system is the major factor which affects the performance of the robot and this control system has a close relationship with at least two of the aspects listed below: the architecture of the robot control system and the servo-control of the joints of the robot. Precise joint-angle control requires a complicated control system.

Most of the robot, especially the multi-variant arm, has, for the sake of reaching a high performance and high accuracy control effect, in general adopted DSP or ARM high processors. For the purpose of control simplicity, we introduced a robot control system based on PIC.

This kind of robot has one CPU board in its body. It is utilized for the real-time controller of whole body motion.

Fig. 3 shows the structure of the humanoid robot controller.

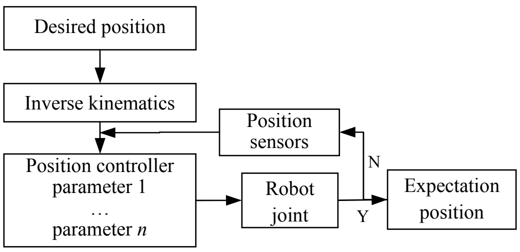

Fig. 3 Structure of the controller

3 Kinematics Analysis

3.1 Forward kinematics

Screw theory is a powerful mathematical tool to solve the kinematics questions of a robot. A simplified robot framework and the model of the arm of this robot are shown in Fig. 4.

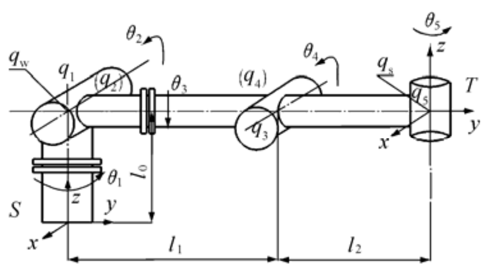

Fig. 4 Model of arm module

We chose one right-handed coordinate frame *S* as a base reference frame, in which the configuration of the rigid body will be expressed. We chose another right-handed coordinate frame *T* as an end reference frame, which rigidly attaches to the body.

Base reference configuration is the robot configuration when the rotated angle $\theta = 0$. The parameter $g_{st}(0)$ has been defined as the *T* and *S* rigid body transformation when the robot lies in the base reference configuration. From Fig. 4, we can obtain the five rotated axes vector ω_i easily. We chose five points q_i on the axes, then calculate the twist ξ_i. According to the Rodrigues formula, the arm circling around the axis of rotation ω for θ angle, then the matrix exponent is

$$\boldsymbol{e}^{\hat{\omega}\theta} = \boldsymbol{I} + \hat{\omega}\sin\theta + \hat{\omega}^2(1-\cos\theta) \quad (1)$$

where $\hat{\boldsymbol{\omega}}$ is the unit vector in the direction of the coordinate axis ω. The twist exponent matrix can be computed by Eq. (2).

$$\boldsymbol{e}^{\hat{\xi}\theta} = \begin{bmatrix} \boldsymbol{e}^{\hat{\omega}\theta} & (\boldsymbol{I} - \boldsymbol{e}^{\hat{\omega}\theta})(\boldsymbol{\omega}\times\boldsymbol{v}) + \boldsymbol{\omega}\boldsymbol{\omega}^{\mathrm{T}}\boldsymbol{v}\theta \\ 0 & 1 \end{bmatrix} \quad (2)$$

The symbol $g_{st}(\theta)$ is taken as the relative configuration of two coordinate frames (between *T* and *S*). The forward kinematics of can be calculated by Eq. (3).

$$g_{st}(\theta) = \boldsymbol{e}^{\hat{\xi}_1\theta_1}\cdots\boldsymbol{e}^{\hat{\xi}_5\theta_5}g_{st}(0) \quad (3)$$

3.2 Inverse kinematics

Inverse kinematics is the basis of robot arm locomotion and control. When studying the prosecution of inverse kinematics operations, researchers try hard to obtain a close radix. A close radix possesses a swift computer rate and high efficiency.

Compared with forward kinematics, the robot inverse joint screw is more difficult. We use Paden-Kahan as reference in our paper[2].

$$g_{st}(\theta) = \boldsymbol{e}^{\hat{\xi}_1\theta_1}\cdots\boldsymbol{e}^{\hat{\xi}_5\theta_5}g_{st}(0) = g_d \quad (4)$$

where g_d is the expected configuration.

When Eq. (4) is right multiplied with $g_{st}^{-1}(0)$ we obtain the formula of exponential mapping as Eq. (5).

$$\boldsymbol{e}^{\hat{\xi}_1\theta_1}\boldsymbol{e}^{\hat{\xi}_2\theta_2}\cdots\boldsymbol{e}^{\hat{\xi}_5\theta_5} = g_d g_{st}^{-1}(0) = g_1 \quad (5)$$

In order to obtain the value of $\theta_i\,(i=1,2,\cdots,5)$, we adopted four steps. The joint corner θ_4 was computed first and then the joint corner θ_5. Two of three remaining joint corners were computed next and, as last, the remaining joint corner was computed.

Both sides of Eq. (5) are multiplied by q_w and subtracted from the point q_s which lies on the fifth axis. Using the characteristic of point-to-point invariability in a rigid body motion process, we obtained the bilateral value of the following equation.

$$\left\|\boldsymbol{e}^{\hat{\xi}_5\theta_5}(\boldsymbol{e}^{\hat{\xi}_4\theta_4}q_w - q_s)\right\| = \left\|\boldsymbol{e}^{\hat{\xi}_4\theta_4}q_w - q_s\right\| = \left\|g_1 q_w - q_s\right\| \quad (6)$$

This question agrees with sub-question 3. The fourth joint angle can be determined by sub-problem 3.

The fourth joint angle is known and Eq. (5) changes to Eq. (7).

$$\boldsymbol{e}^{\hat{\xi}_5\theta_5}q_w = \boldsymbol{e}^{-\hat{\xi}_4\theta_4}g_1 q_w = g_2 \quad (7)$$

This question agrees with sub-question 1 and then the fifth joint angle can be determined by sub-problem 1.

The fourth and fifth joint angles are known and Eq. (5) changes to Eq. (8).

$$\boldsymbol{e}^{\hat{\xi}_1\theta_1}\boldsymbol{e}^{\hat{\xi}_2\theta_2}\boldsymbol{e}^{\hat{\xi}_3\theta_3} = \boldsymbol{e}^{-\hat{\xi}_4\theta_4}\boldsymbol{e}^{-\hat{\xi}_5\theta_5}g_d g_{st}^{-1}(0) = g_3 \quad (8)$$

We selected a point q_3 which lies on the third axis but not on first and second axes.

$$\boldsymbol{e}^{\hat{\xi}_1\theta_1}\boldsymbol{e}^{\hat{\xi}_2\theta_2}q_3 = g_3 q_3$$

This question agrees with sub-question 2 and the first and second joint angle can be obtained by sub-problem 2.

Rearranging Eq. (5), we can obtain Eq. (9).

$$\boldsymbol{e}^{\hat{\xi}_3\theta_3} = \boldsymbol{e}^{-\hat{\xi}_1\theta_1}\boldsymbol{e}^{-\hat{\xi}_2\theta_2}\boldsymbol{e}^{-\hat{\xi}_4\theta_4}\boldsymbol{e}^{-\hat{\xi}_5\theta_5}g_d g_{st}^{-1}(0) = g_4 \quad (9)$$

We selected a point p outside the third axis and entered it into Eq. (9), then: $e^{\hat{\xi}_3\theta_3} p = g_4 p$.

This question agrees with sub-question 1 and the third joint angle can be obtained by sub-problem 1.

It can be seen from the above that the goal of forward kinematics is transforming joint coordinates to end-effector coordinates. On the other hand, the goal of inverse kinematics lies in transforming end-effector coordinates to joint coordinates. The joint angle has multiple possible solutions.

3.3 Jacobian matrix

After knowing the robot parameters and selecting several sets of joint angles θ_i , we can compute the position of the end-effector with the use of MATLAB software.

A Jacobian matrix is one of the important parameters in robot studies. Several approaches can be taken in the process to computer a Jacobian matrix, such as obtaining the Jacobian matrix by kinematics differential vector product but screw theory method is simpler and more convenient.

Because $\boldsymbol{\xi}_i' = \begin{bmatrix} -\omega_i' \times q_i' \\ \omega_i' \end{bmatrix}$.

We computed $\boldsymbol{\xi}_i'$ similarly and obtained the Jacobian matrix as shown in Eq. (10).

$$\boldsymbol{J}_{st}^{s} = \left[\xi_1', \xi_2', \xi_3', \xi_4', \xi_5', \xi_6'\right] \tag{10}$$

4 Conclusions

The features of the robot are summarized as follows: 1) modularization, concurrent design and extension theory methods were adopted in the configuration studies and screw theory was introduced to humanoid robot kinematics analysis; 2) all the motors are installed inside the robot; 3) the framework of the robot is simplified and costs less. Screw theory is a convenient method in the kinematics analysis of humanoid robots. It can be used not only to solve questions of kinematics analysis but can also solve questions of dynamics analysis.

This robot has developed a robotics platform consisting of hardware. We expect that our robot will be able to serve as a research platform for robotics in the near future, because of its feature of open architecture. At present, the prototype of the robot is mounted, but it is not working very well. It will be improved upon in the following aspects: we will develop a six degree of freedom robot arm to substitute the present one and perfect the function of the robot to imitate human beings even better.

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