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ScienceDirect
Human Movement Science 26 (2007) 617–656
HUMAN MOVEMENT SCIENCE
www.elsevier.com/locate/humov

The six determinants of gait and the inverted pendulum analogy: A dynamic walking perspective

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Available online 6 July 2007

Abstract

We examine two prevailing, yet surprisingly contradictory, theories of human walking. The six determinants of gait are kinematic features of gait proposed to minimize the energetic cost of locomotion by reducing the vertical displacement of the body center of mass (COM). The inverted pendulum analogy proposes that it is beneficial for the stance leg to behave like a pendulum, prescribing a more circular arc, rather than a horizontal path, for the COM. Recent literature presents evidence against the six determinants theory, and a simple mathematical analysis shows that a flattened COM trajectory in fact increases muscle work and force requirements. A similar analysis shows that the inverted pendulum fares better, but paradoxically predicts no work or force requirements. The paradox may be resolved through the dynamic walking approach, which refers to periodic gaits produced almost entirely by the dynamics of the limbs alone. Demonstrations include passive dynamic walking machines that descend a gentle slope, and active dynamic walking robots that walk on level ground. Dynamic walking takes advantage of the inverted pendulum mechanism, but requires mechanical work to transition from one pendular stance leg to the next. We show how the step-to-step transition is an unavoidable energetic consequence of the inverted pendulum gait, and gives rise to predictions that are experimentally testable on humans and machines. The dynamic walking approach provides a new perspective, focusing on mechanical work rather than the kinematics or forces of gait. It is helpful for explaining human gait features in a constructive rather than interpretive manner.

Keywords: Biomechanics; Energetics; Passive dynamic walking; Robotics; Human gait; Bipeds

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0167-9457/$ - see front matter
doi:10.1016/j.humov.2007.04.003

1. Introduction

Humans walk through a complex orchestration of muscle forces, joint motions, and neural motor commands. Many of the internal variables that contribute to walking have been measured and quantified over the past century. These include the electromyographic activity, the torques produced by muscles about the joints, the ground reaction forces, the resulting limb motions, and even the metabolic energy cost. This wealth of data requires interpretation and organization through fundamental principles that elucidate the mechanisms of walking. For five decades, two main theories have dominated the study of walking: the six determinants of gait and the inverted pendulum analogy. However, the prevalence of two theories invites the question of whether they co-exist in a complementary fashion or rather contradict each other. A further question is whether two separate theories are in fact necessary, and whether a single theory, either a refinement or replacement of the two, would suffice.

The six determinants of gait theory (Saunders, Inman, & Eberhart, 1953) proposes that a set of kinematic features help to reduce the displacement of the body center of mass (COM). It is based on the premise that vertical and horizontal COM displacements are energetically costly. Motions such as flexion of the knee during the stance phase and rotations of the pelvis are therefore coordinated so as to reduce displacements of the COM from that of a 'compass gait', where the COM travels in a circular arc (see Fig. 1a and b). In contrast, the inverted pendulum theory (e.g., Cavagna & Margaria, 1966; Cavagna, Saibene, & Margaria, 1963) proposes that it is energetically less costly for the stance leg to act like a pendulum and prescribe such an arc. The inverted pendulum theory therefore conflicts with the six determinants theory. It is acceptable for two theories to differ, provided they can be traded off each other for complementary reasons. However, the two theories of walking both serve the same goal of reducing energetic cost, in an opposing rather than complementary fashion.

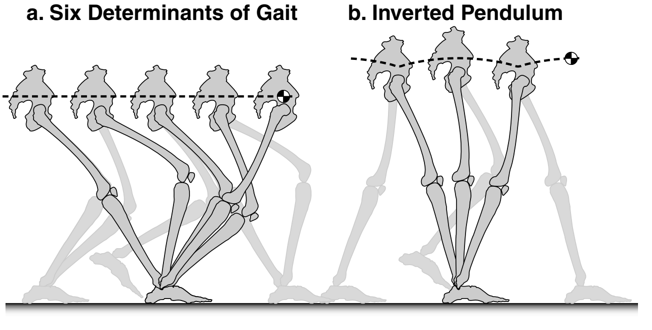

Fig. 1. Two major theories of human walking. (a) The six determinants of gait (Saunders et al., 1953) are motions made by the body to reduce vertical and lateral displacement of the body center of mass (COM). For example, one of the determinants is stance phase knee flexion, which contributes to a flattened COM trajectory. The theory favors a flattened trajectory, as shown. (b) The inverted pendulum analogy states that the stance leg is kept relatively straight during single support, functioning like an inverted pendulum. The COM, located near the hip, travels in a series of arcs prescribed by each single support phase. A related theory is that the swing leg also moves like a pendulum, swinging about the hip. The logical extension of the inverted pendulum theory is that walking can be performed with no muscle actuation, and therefore no energy cost. The two theories offer conflicting opinions regarding the desired COM trajectory.

Further inspection reveals additional cause for concern. The six determinants of gait have practically been accepted as fact for 50 years (Gard & Childress, 2001), appearing in major clinical and scientific textbooks (McMahon, 1984; Perry, 1992; Rose & Gamble, 1994; Whittle, 1996), without being subjected to experimental testing. Recent experiments show that three of the determinants – stance phase knee flexion (Gard & Childress, 1999), and pelvic rotation about the vertical (Kerrigan, Riley, Lelas, & Della Croce, 2001) and fore-aft axes (Gard & Childress, 1997) – actually contribute either negligibly or far less than previously thought to reducing the vertical displacement of the COM. Other experiments testing the premise of the theory show that humans expend more metabolic energy when voluntarily reducing vertical displacement of the COM compared to their normal gait (Gordon, Ferris, & Kuo, 2003; Ortega & Farley, 2005). The 'determinants' are perhaps better viewed as kinematic descriptions of certain gait features whose origins are subject to debate. The inverted pendulum theory fares somewhat better, in that measurements verify that kinetic and gravitational potential energy of the COM fluctuate much as would be expected if the stance leg behaves like an inverted pendulum (Cavagna et al., 1963). A related theory, that the swing leg swings like a (non-inverted) pendulum, is also supported by observations of swing phase timing (Mochon & McMahon, 1980). However, the pendulum theories also present a dilemma: If pendulums can swing freely, why does walking cost any energy at all?

These problems can be addressed through explicit mathematical analysis. A useful theory must make concrete predictions that can be evaluated both theoretically and experimentally. If the qualitative descriptions of the six determinants and inverted pendulum theories are initially vague, they should be clarified and made explicit. Analysis will then reveal whether the theories are sensible, and if so, experimental studies will ultimately provide evidence in support or against. As with the six determinants of gait, the inverted pendulum theory should not be considered immune to testing or refinement. It is no small matter, however, to codify a theory originally stated in an unspecific and qualitative manner. Nor is it always straightforward to subject a specific codification to mathematical analysis. This is because of the many variables that must be orchestrated in the human. The many variables must either be reduced to a smaller number, or specified through a simplifying context such as an optimization principle. Even if these many variables are correctly specified, they may be too numerous to yield testable predictions or contribute to cogent understanding. The rigorous testing of theories requires an appropriate approach that provides both simplification and prediction.

The principles of dynamic walking (Fig. 2) provide just such an approach. Here we use the term 'dynamic walking' to refer to systems in which the passive dynamics of the limbs dominate the motion, with minimal actuation applied to sustain periodic behavior. These principles were originally developed by McGeer (1990a), who also demonstrated passive dynamic walking machines that could descend a gentle slope under gravity power alone. These same principles apply to powered walking on level ground (Kuo, 2002), as demonstrated by several recent walking machines (Collins, Ruina, Tedrake, & Wisse, 2005; Wisse, 2005). Dynamic walking is an extension of the pendulum theory, and has several advantages: It is amenable to mathematical analysis, it can make predictions that can be experimentally tested in human and machine, and the principles are simple enough to be understood. It also offers a potential resolution to the inverted pendulum dilemma, in the form of energy-dissipating collisions between leg and ground. These collisions

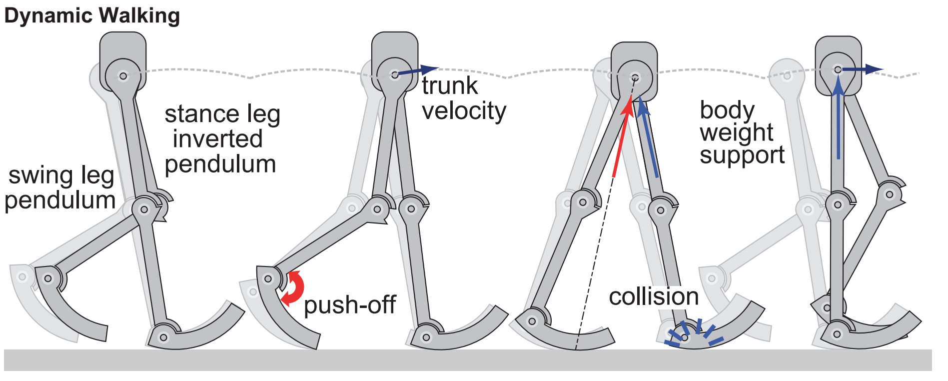

Fig. 2. The principles of dynamic walking yield a periodic walking gait for both unpowered and powered robots. The single support phase can be produced entirely by passive dynamics, with the legs acting like pendulums as in the inverted pendulum theory. However, a key feature of dynamic walking is that there is a collision between the swing leg and ground, dissipating energy. Energy may be restored passively by descending a gentle slope as in passive dynamic walking (McGeer, 1990a), or actively through actuation such as push-off (Kuo, 2002), as demonstrated by a recent robot (Collins et al., 2005). The conditions of push-off and collision can produce initial conditions for the next step, with no need for active stabilization nor for control of prescribed trajectories. With a knee that passively prevents hyperextension (McGeer, 1990b), body weight can also be supported with no actuation. Dynamic walking extends the inverted pendulum theory, indicating that there is an energetic consequence due to the collision, with the transition between pendulum-like steps termed the *step-to-step transition* (Donelan et al., 2002a).

themselves present opportunity for testing, and most importantly, yield distinctive new ways to view and think about human walking.

The purpose of this paper is to codify and evaluate the two main theories of walking, and to contrast them with the principles of dynamic walking. We begin with preliminary principles of locomotion, which shape how dynamical quantities such as force and work are relevant to the biologically important variable, metabolic energy. We then attempt to codify the six determinants and inverted pendulum theories into simple yet explicit mathematical representations, from which predictions can be made and compared with experiment or experience. The principles of dynamic walking are then examined in relation to the conflicts resulting from these comparisons. The dynamic walking approach offers a new perspective for theoretical and experimental examination of locomotion.

2. Preliminary principles of locomotion

The goal of locomotion is obviously to transport the body, but implicit in this goal is the desire to transport the body economically. In humans, metabolic energy is expended for muscles to perform active work, and also to produce force even when no work is performed. During steady motion, positive work is performed with an efficiency (defined as work divided by energy expended) of about 25%. Negative work by muscle is performed with an efficiency of about −120% (Hill, 1938), meaning that positive energy must be expended. The energetic cost of producing isometric muscle force for long durations is largely proportional to the tension-time integral (e.g., Woledge, Curtin, & Homsher, 1985). Unfortunately, these straightforward relationships do not necessarily apply to the complex contraction conditions of walking, where both positive and negative work are performed in short bursts and a variety of shortening speeds. Some negative and positive work may also be performed passively by series elastic elements rather than active contractile elements (Fukunaga et al., 2001). Muscle force is also produced for relatively short rather than long durations, and substantial energy may be expended for the cycling of muscle activation as well as production of force (Doke & Kuo, 2007). The complex conditions of muscle activity during walking therefore exceed the narrow conditions explored thus far through experimentation. Any quantitative approach that attempts to predict muscle force is therefore limited by incomplete characterization of muscle behavior in general, and incomplete data regarding muscle length, velocity, and activation history during walking in specific.

Fortunately, some understanding can be gained through remarkably simple considerations of mechanical work. Rather than focusing on careful quantification of how much work is performed by muscles during walking, we begin with a broader approach. Dynamic walking offers a different perspective than that afforded through more traditional approaches, by considering conceptually why positive work must be performed at all, and how negative work dictates the positive work. Two principles of work provide the basis for examining the two major theories of walking from the dynamic walking perspective.

2.1. Steady locomotion requires equal positive and negative work

In most forms of propulsive locomotion, the human contribution is primarily to positive work, performed against an external resistive load. Examples include pedaling (Fig. 3a) and rowing, in which positive work is performed against drag from water or air. In climbing stairs or a reasonably steep hill, positive work is performed against the resistance of gravity, resulting in an increase in gravitational potential energy. In all of these cases, the environment performs negative work on the COM. The body then performs an equal amount of positive work, so that zero net work is performed on the COM over a cycle. Quantifying the energy expended during such tasks is relatively simple, because the cost is both dominated by and proportional to the positive work performed by muscle. However, understanding of energetic cost is incomplete without consideration of the negative work. For example, one cannot predict the energetic cost of rowing for a particular boat at a particular speed, without a parametric understanding of energetic losses due to inertial, skin, wave, and aerodynamic drag. In fact, serious study of pedaling and rowing is dominated by discussion of these and other dissipative mechanisms. The same is true for transportation by airplane and automobile, where practically every dissipative mechanism is scrutinized, modeled, and parameterized. Once the negative work components are understood, the energetic cost is relatively straightforward, because positive work must be performed in equal amount to the negative work, and work costs energy. Of course, the positive work must be performed efficiently, and there may well be extremely subtle aspects to how it is best performed. Nevertheless, the overall understanding of propulsive locomotion is at least as much in the negative as the positive work.

Contrast this approach with that taken traditionally for human walking. The activation of muscles, the torque and work produced about the joints, and the energetic cost of this effort have been well-characterized and recorded with ever-increasing accuracy. Recent methods allow for the quantification of relative muscle and tendon lengths and their respective contributions to both positive and negative work (e.g., Fukunaga et al.,

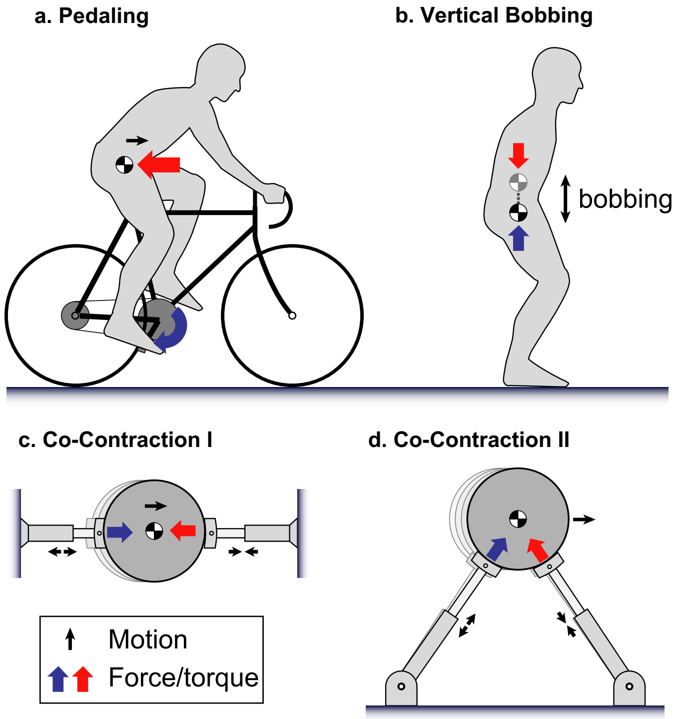

Fig. 3. Simple work tasks illustrate principles of work production. (a) During cycling, the human performs positive work on the crank to offset energy dissipated by the environment, mainly through aerodynamic drag and friction. At steady speed, the rates of positive work performed by muscle and negative work performed by the environment are equal. If drag and friction are negligible, then no work is necessary for locomotion. (b) As a human bobs up and down vertically, the muscles perform positive work against gravity to lift the COM, and then negative work to actively lower it. Gravity performs zero net work on the body over a bobbing cycle, and it is the human who is responsible for both dissipating and restoring energy in equal amounts. (c) A co-contraction task requires simultaneous positive and negative work to transport a spherical mass horizontally (from left to right) at constant speed. Two telescoping actuators push inward with equal and opposite forces to support the mass. The left-most actuator performs positive work against the mass, and the right-most actuator performs negative work. (d) Co-contraction also occurs in a different configuration, where the mass is supported above the actuators, each of which pivots freely about a hinge joint. Both actuators push inward and upward to support the mass, and again one performs positive and the other negative work. Traditional views of walking are often implicitly based on analogies to cycling or bobbing, concerned with work performed to propel or lift the COM. From the dynamic walking perspective, a better analogy might be the co-contraction tasks, because the two legs perform positive and negative work against each other simultaneously during double support.

2001). However, the scientific discussion of walking has traditionally focused primarily on positive work. Positive work is certainly of interest, and the energetic cost of work may have a complex dependence on muscle force, length, and shortening velocity. But relatively little attention has been paid to the negative work. The explanation of walking has therefore traditionally been biased toward positive-work concepts, such as displacing, lifting, accelerating, and propelling the limbs or the body center of mass. However, as with most forms of locomotion, complete understanding of these positive work contributions cannot be achieved without at least equal attention paid to negative work.

2.2. The negative work of walking comes from the body, not the environment

Walking differs substantially from rowing or cycling, in that there is no external load to propel the body against. Because most forms of locomotion involve an external load, one might be tempted to initially assume the same for walking. However, closer inspection reveals that external sources of dissipation are negligible. Aerodynamic drag and friction perform practically no negative work on the body at normal walking speeds, and the ground typically absorbs negligible energy. It stands to reason that negative work is not performed by the external world, but rather by the body itself. Walking is therefore self-resistive, with positive work performed to offset the negative work also performed during the same cycle. From an energetic perspective, it is certainly desirable to minimize the negative work, because that would simultaneously minimize the positive work in a task requiring zero net work. This raises the question of why the negative work should not be reduced to zero, thereby reducing the positive work. It is after all improbable that negative work would be performed arbitrarily. The likely explanation is that biomechanical or task constraints unavoidably require that negative work be performed. It is therefore especially important to understand why this negative work is performed and in what amount, and to understand the underlying constraints responsible for it.

Self-resistive tasks occur commonly in human movement, and their consideration may contribute to the study of walking. One example of a non-locomotory, self-resistive task is bobbing the body up and down (see Fig. 3b), where equal amounts of positive and negative work are performed in succession. Starting from the lower position with the legs bent, the muscles perform positive work to raise the body center of mass against the gravitational load. This work is converted into gravitational potential energy. The next task of lowering the COM therefore requires the steady dissipation of this energy. The muscles perform negative work on the COM to bring the body back to its initial position. Metabolic energy is expended for the positive work, and to a lesser extent, the negative work as well. An obvious means of reducing energetic cost would simply be to remain at one height. If no net change in height is desired over an integral number of cycles, periodic bobbing accomplishes nothing other than to waste energy.

Self-resistance can also occur with positive and negative work performed simultaneously, for example in the horizontal transport of a mass by antagonistic actuators (see Fig. 3c). Here we consider two linear (sliding) actuators, both pushing inward to support a spherical mass. No net work is needed for the mass to move horizontally at constant speed. However, because the actuators are pushing against each other with equal and opposite forces, one actuator performs positive work, and the other negative work. The negative work dissipates the mechanical energy of the positive work. Metabolic energy will be expended to perform positive work and (assuming no means of re-generation) negative work. This may be classified as a form of inter-limb co-activation or co-contraction, extending a term usually applied to antagonists located on opposite sides of a joint to refer to opposite limbs. Co-contraction can also apply to other configurations (see Fig. 3d), for example with the linear actuators both pushing inward and upward simultaneously. Again, the load can be supported as it moves horizontally, with simultaneous positive and negative work performed by the actuators. The actuators function partially as agonists, because they cooperate to resist the force of gravity. However, they also function as antagonists, with the horizontal component of forces in opposition. Most researchers view co-contraction as energetically unfavorable, employed to satisfy task constraints at the expense of energy expenditure. For example, co-contraction might enhance joint stiffness or otherwise improve the stability of a task. Co-contraction is also difficult to avoid completely; most movements have some degree of inter-limb or inter-joint co-activation, as a consequence of the limited directions of muscle force (Kuo, 1994, 2000). In the cases considered here, inter-limb co-contraction is necessitated by the requirement to support the load and the available directions of force application.

Each of the above tasks could potentially be analogous to bipedal walking. Gait research has often employed concepts such as propelling or lifting the COM, perhaps analogous to pedaling against an external load or bobbing up and down, respectively. Propulsion certainly becomes a critical goal when the environment dissipates energy. However, it applies less well to walking, where only the body dissipates energy. The cost of propulsion might sensibly be reduced simply by choosing to dissipate less energy, leading to the question of why humans dissipate energy at all. As discussed above, a study of propulsion is incomplete without an understanding of why and how the negative work is performed in the first place. Vertical bobbing also offers an initially attractive means of explaining the cost of walking. Raising and lowering the COM during walking is a form of bobbing that appears to underlie the six determinants of gait theory. The analogy to bobbing explains why the theory claims that vertical displacement of the COM is undesirable. However, this premise leads to the question of why humans walk with any vertical motion at all. As with vertical bobbing, it would seem more economical to keep the COM at a fixed height. Perhaps task constraints prevent the achievement of a level path for the COM. But if task constraints do necessitate raising and lowering of the COM, this leads naturally to the question of what the constraints are and why some displacement must occur. The propulsion and bobbing analogies, despite their apparent applicability to walking, raise more questions than they answer.

A possible explanation for these issues is simply that a different analogy may apply better to human walking. We propose that inter-limb co-contraction in fact serves as a superior analogy compared to bobbing and propulsion, with the human legs during double support taking the place of the sliding actuators used in the example of Fig. 3d. The analogy applies well to human walking because the legs perform simultaneous positive and negative work during double support (Donelan, Kram, & Kuo, 2002b), with substantial metabolic cost (Donelan, Kram, & Kuo, 2002a). It is also apt because co-contraction is generally understood to be energetically costly and to be performed only when task constraints make it otherwise favorable. There nevertheless remain the questions of why the negative work cannot be avoided, and what task constraints require that it be performed. This question may be addressed by considering the work performed on the COM during walking. Simple models of locomotion will prove useful in highlighting the costs associated with different types of walking, and the task constraints that favor inter-limb co-contraction.

3. Simple models of walking

We next consider simple models of the leg forces acting on the COM during walking. We model the COM as a point mass located at the hip, assuming it to be the primary load during walking. This allows for calculation of the potential costs of transporting the COM alone. We therefore neglect the mass of the legs and rotational inertia of the trunk. These may in fact present significant load as well, particularly if the trunk rotates substantially, but accounting for these loads only adds to the overall theoretical cost. The simple models therefore yield lower bounds rather than actual estimates of the theoretical costs of locomotion. Nevertheless, the six determinants of gait, inverted pendulum, and step-to-step transition theories differ quite dramatically in their predictions, providing insight into their relative advantages and disadvantages.

Each theory will be considered in terms of the positive work rate (WR) and metabolic rate (MR) needed to walk at a nominal speed. Average positive work rate is defined as the positive work performed over a step multiplied by step frequency. Metabolic rate is the average rate at which metabolic energy is expended, expressed in similar units of power. Work must be performed at a metabolic cost, and so work (performed at an assumed efficiency) can predict a lower bound on MR. Metabolic rate, however, also depends on other factors that need not be attributable to work (Woledge et al., 1985), and will therefore often exceed the prediction based on WR alone. We will use both quantities in dimensionless form, normalizing by body weight multiplied by $(gL)^{0.5}$, where g is gravitational acceleration and L is leg length. This normalization helps to account for differences in energy expenditure due to body size, with $(gL)^{0.5}$ recognizable as the term also used to normalize speed of locomotion (e.g., McMahon, 1984; McGeer, 1990a). Dimensionless metabolic rate is also equal to the dimensionless cost of transport multiplied by the dimensionless speed, where cost of transport is defined as energy expended divided by body weight and distance traveled (Tucker, 1975). During normal walking at a nominal speed of $1.25\ \mathrm{m\,s^{-1}}$, a 66 kg human typically has a net metabolic rate of about 150 W (where 'net' refers the total energetic cost subtracting that for quiet standing), or dimensionless MR = 0.076 (Donelan et al., 2002a).

3.1. The six determinants of gait predict very high energy expenditure

The premise of the six determinants of gait is that it is undesirable to displace the body center of mass, especially in the vertical direction (Saunders et al., 1953). The implication is that displacing the center of mass is costly, because work must be performed to accelerate it any direction (even fore-aft), or lift it vertically against gravity. This argument against COM displacement would be most sensible if the COM were to slide atop a smooth horizontal surface, or if it were carried atop a frictionless rolling cart. These cases would allow the COM to translate at constant speed with no need for work. Work might instead be needed to displace the COM from side to side, or to lift it from the horizontal path, and that work would entail an energetic cost. Appealing as this scenario might apply, however, the COM does not slide horizontally but is instead carried by two legs. Saunders et al. (1953) state that translation in a straight line is "quite impossible by means of bipedal gait. The next most economical method would be ... a sinusoidal pathway of low amplitude ... " Here we consider a simple model in which a level path is indeed possible; this may alternatively be interpreted as a sinusoidal pathway of vanishingly low amplitude.

The level path may be analyzed quantitatively in terms of muscle work. The mechanics are amenable to free body diagram analysis, so that the implications may be made explicit. A level path places constraints on the conditions for single and double support. During single support, the COM can only be translated perfectly horizontally if the stance leg's effective length (distance from center of pressure of ground contact to the COM) is continuously adjusted as it pivots about the foot. The horizontal path may be maintained during double support by co-ordinating the lengths of both legs. These constraints require that the legs be bent through most of a stride, with serious energetic consequences. During single support, a bent leg presents a significant moment arm for the force of body weight about the knee, implying the need for a high extensor torque about the knee and substantial work performed on the COM. During double support, body weight may be distributed between the two legs, both producing force in extension. But the forward path of the COM also means that the trailing leg must perform positive work throughout double support, and the leading leg negative work. Practically every aspect of such a path requires high joint torques and substantial work, both with a high energetic cost.

We conduct the analysis of the six determinants trajectory with a simplified representation of walking in the sagittal plane, in which the COM is supported by two legs of negligible mass (Fig. 4a). We assume that the COM travels at a constant height h above ground, which requires that the legs always produce vertical force equal to body weight. The conditions of ground contact also place an important constraint on the direction of leg forces: they must be directed along the legs themselves, from the center of pressure of ground contact to the center of mass. This is because neither leg can exert a torque about the center of pressure of ground contact, and the legs have negligible mass. During double support, we also assume that the fore-aft acceleration is zero, to avoid unnecessary work to accelerate and decelerate the COM. These constraints are sufficient to fully determine the leg forces as a function of the relative durations of single and double support (see details in Appendix A). Analysis shows that the least cost is achieved when double support is made instantaneous, yielding a minimum work rate of

$$\mathrm{WR} = \frac{s^2 f}{4\sqrt{4 - s^2}}, \tag{1}$$

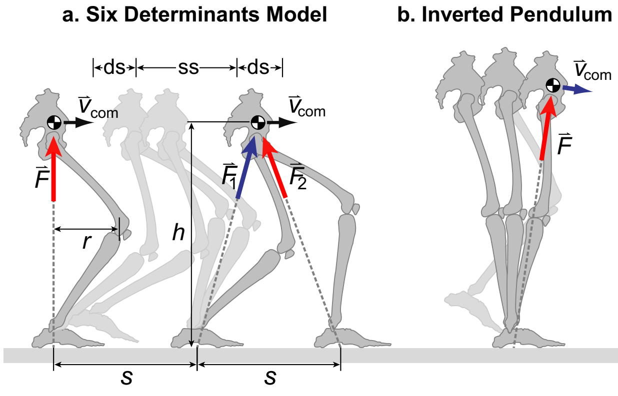

Fig. 4. Simple models of walking, with forces from each leg acting on the COM. (a) In the six determinants theory, the COM moves in a level path at constant velocity $\vec{v}_{com}$. The stance leg produces a force $\vec{F}$ directed at the COM from the center of pressure on the ground. This force both supports body weight and performs positive and negative work to maintain the level COM path. During double support (indicated by 'ds', with 'ss' standing for single support), both legs produce force (denoted by $\vec{F}_1$ and $\vec{F}_2$), subject to the same constraints. The flattened trajectory requires a great deal of work during both single and double support. The bent-legged configuration also requires high knee torques to support body weight. The geometric variables are the height h of the COM, the step length s, and the moment arm of body weight about the knee r, all normalized by leg length. (b) In the inverted pendulum theory, the stance leg is relatively straight and supports the COM. The COM moves in a pendular arc, with no mechanical work needed to sustain the motion.

where s and f are the normalized step length and step frequency, respectively (with leg length L and pendular frequency $g^{0.5}L^{-0.5}$ acting as the respective normalization factors). As with any zero-work task, there is an equal amount of negative work performed. During single support, this negative work is performed by the stance leg prior to mid-stance, followed by positive work after mid-stance. During double support, negative work is performed by the leading leg, simultaneous with the positive work of the trailing leg. The level COM path therefore requires positive and negative work that cancel both in succession (similar to bobbing in Fig. 3b) and simultaneously through inter-limb co-contraction (similar to Fig. 3d).

This mechanical work rate can also predict a minimum metabolic rate. At a nominal walking speed of 1.25, humans prefer a normalized step length of about 0.7. Assuming that positive work is performed at an efficiency of 25% and that negative work is performed at −120%, the resulting energetic cost of transport is $\mathrm{MR} = 0.18$. This is more than double that observed in humans. Of course, not only do humans walk with lower energetic cost, they also clearly do not transport the COM on a level path. The simple model merely illustrates the obvious fact that a level path is energetically uneconomical. Another cost not included in this calculation is for supporting body weight. The COM's level path necessitates that the legs be bent for almost all of step, so that body weight is supported with a substantial moment arm r of gravitational force about the knee. This requires substantial knee torque, which may come at a high energetic cost without even considering work.

These costs could be reduced significantly by taking shorter steps. The predicted metabolic rate approaches zero as step length is reduced to zero (taking the limit of Eq. (1) while keeping walking speed constant). Short steps allow the leg forces to be more vertical, so that there is less cancellation in the fore-aft components, and therefore less inter-limb co-contraction and lower work rate. Another advantage is that the COM can be carried at a greater height and therefore straighter legs, reducing the moment arm r with which body weight is supported. Shorter steps, however, come with disadvantages. They require a higher step frequency to maintain the same speed. Higher step frequency implies that the legs must be moved quickly in the transition between double support phases. The simple model above (Eq. (2)) did not consider such costs, which appear high enough to make it costly to walk at higher step frequencies than normally preferred. Even though the work performed during double support can theoretically be reduced to zero with shorter steps, this is offset by the high cost of faster steps (Doke, Donelan, & Kuo, 2005).

Human experiments also confirm the high cost of walking with a flatter COM trajectory. When given visual feedback of vertical pelvis position and asked to minimize the displacement, humans do not generally produce the level path assumed in the simple model. Their gait bears a greater resemblance normal walking, albeit with the legs bent considerably at mid-stance. A more level path appears too difficult for normal humans to achieve easily. Our measurements (Gordon et al., 2003) of normal adult humans ($N = 10$) showed that they managed to reduce their COM vertical displacement by 39%, with a more than doubling of net metabolic rate, increasing by 113%. These are corroborated by similar results reported by others (Ortega & Farley, 2005). These data agree reasonably well with the high cost predicted by the simple model for walking with a level COM path, even though human subjects cannot produce a perfectly level path. Neither the model nor experiments yield support for the six determinants theory.

3.2. The inverted pendulum model predicts no energy expenditure

A simple model of the inverted pendulum theory (Fig. 4b) yields an opposing perspective to the six determinants of gait. In the simple pendulum, the COM is supported by a massless leg during single support, with the ground reaction force $\vec{F}$ directed from the center of pressure to the COM. The COM velocity $\vec{v}_{\mathrm{com}}$ is directed perpendicular to the ground reaction force. No work is performed on the COM, which may be transported along the pendular arc with no energy input. Conservation of mechanical energy applies not only to the simple pendulum (Mochon & McMahon, 1980), but to other varieties of pendulum with mass distributed along the rod as well. In these cases, the ground reaction force need not be perpendicular to COM velocity – a convenience for illustrating the work performed on the COM – but again no work need be performed to move along the pendular arc. This is in direct conflict with the six determinants theory, which prefers a flattened trajectory.

In the context of human motion, the pendulum analogy for human walking poses a paradox regarding energy expenditure. If the swing leg moves like a pendulum (Mochon & McMahon, 1980), and the stance leg like an inverted pendulum, the energy-conserving properties of both require no work. Another advantage of the inverted pendulum is that, with the stance leg relatively straight, little knee extensor torque is needed to support body weight. The pendulum analogy therefore offers multiple explanations for why walking should cost no energy whatsoever, and not why it costs any energy at all. It is therefore necessary to consider how walking deviates from pendulum behavior, and whether this deviation might cost energy.

One possible explanation for the actual energetic cost is that the stance leg does not behave passively. Instead, it may act as a forced pendulum, with muscle work performed to accelerate and decelerate it. However, given the possibility of acting passively with no energy expenditure, it would make little sense for humans to prefer a more expensive alternative. We would also expect that some slow walking speed would exist for which there would be zero energetic cost. The metabolic rate is in fact quite substantial for all walking speeds. Another possibility is that energy is expended to produce the muscle force needed to keep the leg rigid. However, the configuration of the leg, with a passive mechanical stop preventing knee hyperextension, would not appear to require high muscle forces to maintain extension. Nor would isometric force production explain why actual energy expenditure increases with walking speed. Neither explanation appears to account for the energetic cost of walking.

The inverted pendulum nonetheless remains appealing as a model for walking. Observations agree that the stance leg remains fairly straight during single support, and the exchange of potential and kinetic energy during this period also agrees well with what would be expected of a passive inverted pendulum. However, the same model must also be considered incomplete, because it offers a strategy for reducing energy expenditure to zero. Taken to its logical conclusion, the inverted pendulum theory prescribes a strategy not taken by humans, and explains none of the actual energy cost of walking.

4. Mechanics and energetics of dynamic walking

Another approach to human gait is to consider the affordance of an entire gait cycle with minimal actuation. The principles of dynamic walking were initially developed for the construction of gait in legged machines (McGeer, 1990a), without regard to muscles, joint motions, nor even consideration for empirical human behavior. This would seem to offer little advantage over a human-centered approach, but by dispensing with interpretations of why humans act in a particular way, it focuses instead on what is fundamentally necessary for a gait to exist. The constructive approach is also amenable to experimental testing, by both machines designed to demonstrate the principles and by experiments performed on human subjects.

Dynamic walking is based on and extends the inverted pendulum theory. As with the pendulum model, the stance leg can swing freely like an inverted pendulum, with the pelvis moving through an arc. Body weight can be supported passively by a stop located at the knee preventing hyperextension. The leg will remain extended against the knee stop throughout the stance phase, provided that an extensor moment is applied about the knee and against the stop. McGeer (1990b) showed that, by designing the feet to face forward from the legs, such an extensor moment could be supplied passively. Similarly, the swing leg's motion can be driven entirely by pendular dynamics. With human-like proportions for the relative lengths of thigh and shank, the natural motion can easily provide sufficient knee flexion for the swing foot to clear the ground during mid-stance. The combined motion of the stance and swing legs, as described thus far, requires no active work, nor even active force production. The paradox of the inverted pendulum is therefore not resolved by single support alone. Where dynamic walking differs from the inverted pendulum theory is in the completion of an entire gait cycle. The inverted pendulum theory models single support, with little consideration for what happens between single support phases. The double support phase has traditionally proven difficult to model, because the legs form a closed-loop kinematic chain, with no simple and obvious mechanical analogy such as the inverted pendulum. The dynamic walking approach, however, resolves this difficulty by treating the step-to-step transition as analogous to a collision. We next consider how the step-to-step transition and other features of dynamic walking impact the mechanics and energetics of human walking.

4.1. Step-to-step transitions require energy to perform mechanical work

In dynamic walking, the energy-conserving motion of single support is interrupted by the collision of the swing leg with ground (see Fig. 5a). The collision brings the foot to a sudden stop, with energy dissipated inelastically by the impact. Although the collision may appear to occur locally at the point of contact, the actual energy dissipation may occur throughout the leg and even in other parts of the body. (Some energy may also be dissipated through the noise of the impact and deformation of the ground, but in a much smaller amount.) Aside from the energy loss, the other major dynamical effect of the collision is to change the velocities of the legs and the COM. The velocity change is necessitated by the exchange of stance legs, where the pendular arc described by one stance leg must give way to another arc prescribed by the other leg. In most examples of dynamic walking to date, the collision is modeled inelastically, with energy dissipated through negative work performed by the body. The negative work can be performed actively through muscle, through soft tissue deformations, and in multiple locations throughout the body (Kuo, Donelan, & Ruina, 2005), the details of which cannot be predicted from first principles. It is, however, a simple matter to predict the velocity and energy changes for the COM. These computations are made using the assumption of inelasticity, the constraints

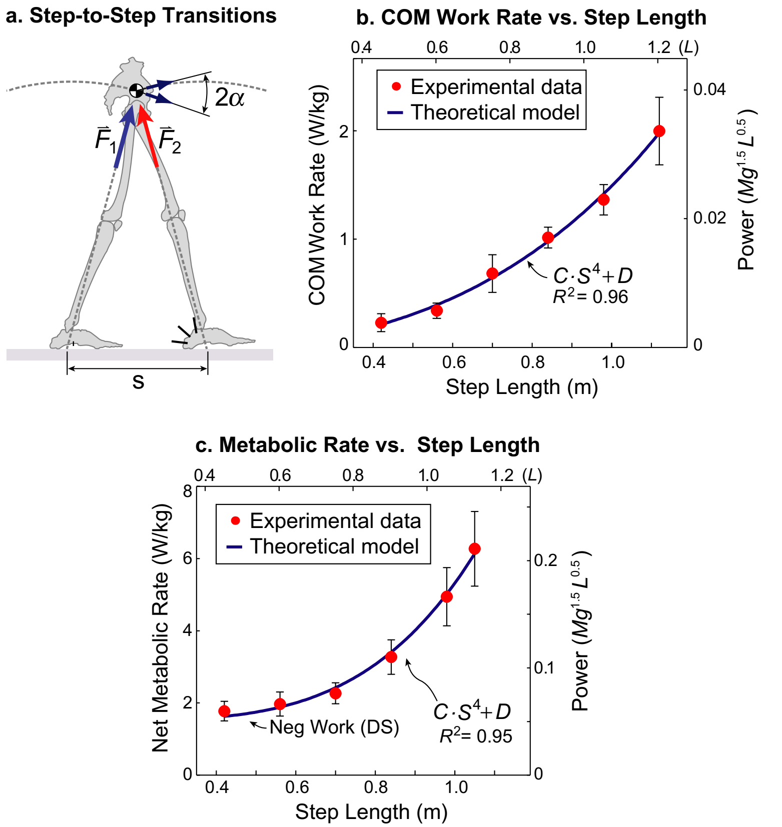

Fig. 5. (a) A major energetic cost in dynamic walking is for the work performed in step-to-step transitions (Donelan et al., 2002a), which occur mainly during double support. The two leg forces $\vec{F}_1$ and $\vec{F}_2$ redirect the COM velocity from a downward and forward direction to an upward and forward direction, sweeping an angle 2α. Negative work is performed on the COM in the collision between leading leg and ground, and the energy lost may be restored through positive work by the trailing leg. The amount of redirection increases sharply with the normalized step length s, so that the average rate of positive work on the COM increases with s^4. (b) Experimental measurements of humans walking at constant step frequency but increasing step length support the simple model predictions. The data show the work performed on the COM during double support. (c) Rate of metabolic energy expenditure also increases with s^4 and in proportion to COM work rate. Data shown are plotted in normalized form (see right-hand and top axes for normalization factors), with axes converted to more familiar dimensional units (left-hand and bottom axes) by multiplying by the appropriate averaged normalization factors.

imposed by the stance leg conditions before and after the step-to-step transition, and work-energy principles.

A simple model reveals the energetic consequences of the collision. The simplest model of powered walking includes a point mass for the COM located at the hip, along with massless legs, at the end of which are point mass feet. The feet are modeled with infinitesimal mass compared to the body COM, but this is sufficient for the leg to swing like a pendulum, where the period is determined by its length. With point feet, the stance leg prescribes a compass gait, with the COM velocity perpendicular to the leg as in the inverted pendulum model (Fig. 4b). Denoting the angle between the legs at the end of single support as the quantity 2α, the COM velocity must also undergo a direction change 2α. In passive dynamic walking, the leading leg performs negative work on the COM and reduces the magnitude of COM velocity, so that the next single support phase begins with less speed than the preceding one. Gravity then supplies the necessary energy to restore speed as the machine descends a slope (Garcia, Chatterjee, Ruina, & Coleman, 1998). In active dynamic walking, the energy loss may be reduced by 75% by applying a push-off impulse immediately prior to the collision (Kuo, 2002). The reduction in loss occurs because the push-off reduces the velocity of collision. The mechanical cost of transport of this push-off powered gait is (see Appendix A for details)

$$\mathrm{WR} = \frac{1}{8} f^3 s^4. \tag{2}$$

The impulsive nature of push-off and collision means that the two impulses can theoretically occur in quick, and theoretically instant, succession. However, a practical push-off and collision could also occur over a longer duration (Donelan et al., 2002a), and even overlap in an inter-limb co-contraction. Overlap of push-off and collision has the disadvantage of requiring additional work, because the two legs' opposing forces will then act across a non-zero displacement. A short duration for the step-to-step transition is theoretically advantageous, because it also entails a short COM displacement and less co-contraction work. Step-to-step transition work is minimized with the shortest practical duration.

The double support phases of the six determinants (Eq. (1)) and step-to-step transition models (Eq. (2)) are actually governed by the same work principles. For double support phases of non-zero duration, both models require that simultaneous positive and negative work be performed by the two legs. The difference is that the six determinants model requires no net change in COM velocity from one step to the next, and so co-contraction is associated only with the amount of COM displacement during double support rather than redirection. The work may in fact be reduced to zero by making double support instantaneous (as is the case for Eq. (1)). In the dynamic walking model, the COM velocity must be redirected regardless of the duration or displacement of the COM, with a consequently higher cost for the step-to-step transition.

The total costs of single and double support are such that the dynamic walking gait is far more economical. Single support requires a great deal of work for the level path of the six determinants model, and no work for the dynamic walking model. For the same nominal gait and muscle efficiencies assumed previously, the resulting minimum energetic rate for dynamic walking is $\mathrm{MR} = 0.027$, less than one-sixth that predicted for the level path gait. A pendular gait, even with the consequence of step-to-step transitions, requires far less mechanical work than the gait favored by the six determinants theory.

The theoretical cost of step-to-step transitions may be compared with human energy expenditure. In absolute terms, the model underestimates the actual cost of human walking ($\mathrm{MR} = 0.076$) to a considerable degree. But as an estimated theoretical lower bound, it does not include many other possible costs that surely contribute to human walking. For example, the simple model achieves high economy with a push-off that occurs impulsively, whereas human push-off must occur with a limited force magnitude and over a non-zero time duration. Non-impulsive push-off therefore incurs a substantial energetic penalty compared to the model. These differences between model and human are difficult to predict from first principles, nor is the simple model's lower bound prediction intended to predict actual energy expenditure in an absolute manner. The simple model is, however, intended to predict the general nature of energetic cost, and especially its dependency on gait variables such as step length, step frequency, and walking speed.

One comparison between trends predicted by the model and those observed in humans may be made in terms of step length. Keeping step frequency f fixed and treating step length s as the independent variable, Eq. (2) predicts that WR will increase in proportion to s^4. This work is also predicted to require proportional metabolic energy, with MR also proportional to s^4. Measurements of normal adult human subjects ($N = 9$) confirm both of these predictions (Donelan et al., 2002a). The average rate of work, computed from the positive work performed by the trailing leg on the COM during double support (or equivalently, the magnitude of negative work by the leading leg) increased approximately with s^4 (Fig. 5b). The corresponding net metabolic rate also increased approximately with s^4 (Fig. 5c), or in proportion to work rate. Step-to-step transitions require substantial metabolic energy expenditure, increasing sharply with step length.

Step-to-step transition costs may theoretically be reduced by taking shorter steps (Kuo, 2002, 2001). Shorter steps allow the COM velocity to be redirected by smaller amounts. In the limit of very short steps, the arcs prescribed by the inverted pendulum become shorter and flatter, approaching the same level path of the six determinants theory. Eq. (2) indicates that MR depends more on step length than on step frequency, so that in the limit of very short but fast steps – keeping walking speed constant – energetic cost will approach zero, just as with the six determinants theory. Taken to its logical conclusion, the theory of step-to-step transitions presents a similar difficulty, in that it favors shorter and faster steps than humans actually prefer. There must clearly be another energetic cost for high step frequencies that is separate from step-to-step transitions.

The analysis above employs extremely simplified assumptions that help to illustrate the principles of step-to-step collisions. As such, the model with point masses would not be expected to accurately predict actual collision losses. However, practical experience with walking machines shows that the addition of human-like mass distribution makes negligible difference to the qualitative behavior of collisions. The addition of knees (McGeer, 1990b) or side-to-side rocking motion (Donelan, Shipman, Kram, & Kuo, 2004; Kuo, 1999) to the model may increase the energetic costs, but only by changing a proportionality factor in Eq. (2) without substantially altering the fundamental dependencies on step length and frequency. This explains why a human, with quite different mass distribution from the simple model, will still expend energy with MR proportional to s^4. The existence of physical walking machines also helps to assure that the dynamic walking principles are not somehow incomplete nor based on entirely unrealistic assumptions. Had physical walking machines been constructed to demonstrate the two major theories of walking, we suspect that these machines would either incur very high energetic costs (in the case of six determinants theory), or be unable to complete successive steps (inverted pendulum theory). Experimental robotics can be useful for testing and demonstrating basic principles of bipedal locomotion.

4.2. Forced leg motion also requires energy, not necessarily for work

Step-to-step transitions represent a major cost of walking, but by no means the only cost. The analysis thus far has considered only work performed on the COM, assuming massless legs. However, human legs do have appreciable mass, and the forced motion of this mass relative to the torso might also require energy expenditure, increasing as a function of step frequency. This cost for forced leg motion would then act as a trade-off against step-to-step transition costs, so that the total cost would be minimized at an intermediate combination of step length and frequency rather than the extremes favored by either cost alone (Kuo, 2001). Such a cost is corroborated by data showing that energy expenditure increases sharply with step frequency, even when step length is kept fixed (Atzler & Herbst, 1927). Data from birds also indicates a substantial energetic cost for leg motion (Marsh, Ellerby, Carr, Henry, & Buchanan, 2004). Although this cost is simple to justify in general terms, it is another matter to attribute to it a proper quantitative mechanism.

The advantage of taking faster steps may be demonstrated with a torsional hip spring acting between the legs (Kuo, 2002). The spring produces equal and opposite torques on the legs, proportional to the angle between the legs. Whereas increasing push-off work alone produces longer steps at approximately constant step frequency, the effect of increasing hip spring stiffness alone is to produce faster steps with only a slight negative effect on step length. For the same amount of push-off, a hip spring will produce a gait with higher step frequency and nearly the same step length, for net gain in walking speed. A spring can therefore enhance the pendular swinging action, and therefore walking speed, without requiring additional work. A hip spring is also theoretically far better than a motor that performs net work at the hip. Work at the hip is another means of powering gait, but it is uneconomical compared to a properly-timed push-off, because it does not reduce the collision cost. The spring in concert with push-off is preferable because the collision is reduced, and faster steps may be taken for the same amount of work.

It may also be advantageous to emulate a hip spring with active muscles, even if doing so comes at an energetic cost. Passive joint stiffness at the human hip is not appreciable enough to affect the leg's natural frequency of swinging (Doke et al., 2005). Humans must therefore rely on muscles to speed leg motion. Net work at the hip may be avoided by activating the muscles so that they produce the same torque per angular displacement (and therefore the same work) as a spring. This activity, however, would still be expected to cost energy. Net economy may be improved by combining push-off with active emulation of a hip spring in an optimal manner. This depends on the energetic cost of emulating a spring.

One obvious candidate for the cost of emulating a spring is the within-cycle work performed on the leg. Even though zero net work is required to move the legs through a complete cycle, positive and negative work must be performed on the leg within a cycle, somewhat analogous to the bobbing motion (Fig. 3b). There may be a metabolic cost associated with this within-cycle work. Keeping the amplitude of leg motion fixed, the rate of within-cycle, positive work performed on the leg is predicted to increase with f^3s^2 (Kuo, 2001), where f is the normalized frequency of swinging. This cost, however, would not appear to explain the actual, energetically optimum combination of step length and frequency preferred by humans (Elftman, 1966; Grieve & Gear, 1966). Summed with step-to-step transition costs, it would tend to favor a smaller change in step length as a function of speed than is observed (Kuo, 2001).

Further inspection reveals why work might not explain the energetic cost of moving the legs quickly. Forced back-and-forth motion of the legs can in fact be produced with no active work, if series elastic elements produce it passively (Kuo, 2002). Series elasticity, primarily attributable to tendon, is theoretically capable of performing all of the negative and then positive work on the leg, storing and returning the energy elastically. Active contractile work may be avoided by activating muscle in brief bursts of isometric force, with timing and duration appropriate to the frequency of motion. Muscles must produce force to move the legs, whether or not they also perform work. The most economical strategy might therefore be to produce force alone, relying on tendons to perform the work passively. The energetic cost of such a strategy would then be dominated by the cost of producing force rather than performing work.

A proposed cost for producing muscle force requires both a mathematical description and a plausible scientific explanation. The mathematical description may be found by deducing the cost that, when added to step-to-step transition costs, yields a total whose optimum matches that observed in humans. A good agreement with the metabolic optimum is in fact given by a metabolic rate proportional to f^4s which we term a cost of producing cyclic force (Doke & Kuo, 2007). Such a cost, unusual as it may seem, can potentially be explained as follows. Energy is expended by muscle primarily for crossbridge attachment (actomyosin ATPase), and for calcium transport (sarcoplasmic reticulum ATPase). Crossbridge attachment usually dominates the overall energetic cost when muscles perform substantial work. This may remain the case even when muscle is activated isometrically, because crossbridges continue to attach and detach if the force is produced for long durations, yielding a cost proportion to the familiar tension-time integral. However, when muscle force is produced for short durations, calcium transport may dominate (Bergstrom & Hultman, 1988; Hogan, Ingham, & Kurdak, 1998). We have proposed that short periodic bursts of muscle force may be rate-limited by crossbridge attachment and detachment (Doke & Kuo, 2007). To produce the same amount of force at higher frequencies, it may become necessary to compensate by releasing and then pumping more calcium, even though the crossbridge attachments does not increase. The cost per contraction would be expected to be proportional to muscle force and to the frequency (i.e. inverse of burst duration) of force production. The MR, or average cost per time, would be expected to increase with f^4s, as needed to produce the desired overall metabolic optimum.

The hypothesized cost of cyclic force production can be tested by independent experiment. Attributing this cost primarily to the activation of hip muscles, the cost may be tested by isolated swinging of the leg. Substituting a constant amplitude of swinging for step length (and frequency of swinging for step frequency), the predicted metabolic rate would be proportional to f^4. We measured human subjects ($N = 12$) performing isolated swinging of a single leg (Doke et al., 2005), held straight by a lightweight splint. The conditions were roughly matched to walking, with approximately the same range of hip torque and angular displacement observed during walking (Fig. 6b). The resulting metabolic rate increased approximately with f^4, as predicted. A subsequent study further showed that the energetic cost cannot be explained by the work performed on the leg (Doke & Kuo, 2007). We tested the cost of swinging the leg at an amplitude decreasing with frequency, such that the rate of work on the leg was held constant. The resulting metabolic rate increased as predicted by the cyclic force hypothesis (f^4s, with s decreasing with f to keep work rate fixed), rather than remaining constant as would be predicted for work. One substantial component of the energetic cost of moving the leg back and forth is therefore not explained by work.

Isolated swinging of the leg, while demonstrating a substantial cost for cyclic force production, differs in several respects from normal walking. First, zero work is performed on the leg over a cycle of leg swinging, whereas some net positive work is performed at the hip

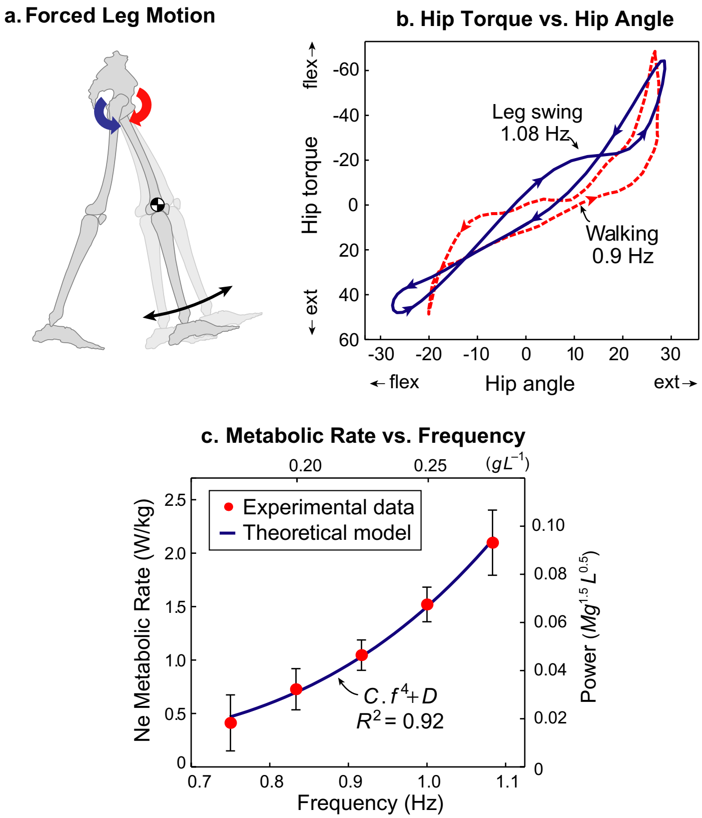

Fig. 6. Another important energetic cost in walking is for forced motion of the legs relative to the body (Doke et al., 2005). (a) A simple pendulum model of moving a single leg relative to the body shows that positive and negative work must be performed on the leg within each cycle, even though no net work is performed over an entire cycle. Much of this work can potentially be performed passively by elastic tendon, but active muscle force is nonetheless necessary to support this motion. (b) The energetic cost may be estimated from an isolated leg swinging experiment, in which the leg is moved about the hip with roughly the same torque and angle amplitudes as walking. A predicted cost, dominated by a hypothesized cost for production of muscle force in short bursts, increases with f^4, treating f as the frequency of swinging, keeping amplitude fixed. (c) Experimental measurements show that the cost of isolated leg swinging increases sharply, and in approximate agreement with the f^4 prediction.

in walking. Second, leg motion during walking involves motion about both the hip and knee. Third, both legs move during walking. The effect of these differences may be evaluated as follows. Net positive work at the hip would be expected to add to the cost of producing the same range of torque and amplitude but without work. Production of force or work about the knee would also be expected to add to the cost of forcing the hip, as might motion of two legs as opposed to one. In other words, all of the differences with walking would be expected to increase the actual cost of forced leg motion during walking beyond that estimated from isolated leg swinging. The results summarized here indicate that, at minimum, there is a substantial energetic cost to moving the legs, irrespective of work. They also indicate a possible mechanism – calcium transport – that can be tested independently of walking.

The dynamic walking approach suggests a different perspective on the contributions to the cost of walking. The dynamic walking analysis separates costs into those for performing work on the COM and those for moving the legs relative to the body. The work performed on the COM during both single and double support appears to contribute to the overall energetic cost of walking, and the dynamic walking approach need not separate the two (even though the majority of the work is performed during double support). The second cost is based the torques needed to force the legs to move relative to the body, also without the need to distinguish between single and double support phases. A more conventional approach is to separate walking into distinct single and double support phases, and to examine the work performed at each joint. The dynamic walking approach favors a different decomposition, focusing on work performed on the COM along the legs, and forces exerted by the body to move the legs back and forth. This decomposition focuses on only two relatively simple costs, and provides a context for examining the action at each joint.

4.3. Dynamic walking yields stable limit cycles

One of the most interesting aspects of dynamic walking is that the heelstrike collision can automatically produce velocity changes that give rise to another step that repeats the previous one. In passive dynamic walking down a slope, the energy lost in the collision is restored by gravity. In active dynamic walking, positive work must be performed to offset the energy lost in the collision. In both cases, the natural dynamics admit a stance phase and collision that reproduce the original initial conditions. In the terminology of dynamical systems, this periodic gait is referred to as a *limit cycle*. It is a special feature of dynamic walking that these limit cycles can arise naturally, without need for intervention from a control system or even actuators. In fact, it is typical that multiple limit cycles exist, representing different gaits or variations of a gait. Of course, an arbitrary assemblage of limbs and joints would hardly be expected to admit limit cycles. Walking occurs within a constrained range of geometric proportions and mass distributions, and human-like proportions fit within that range. However, once a single limit cycle is discovered, more can usually be found for variations of a model. This makes it straightforward to study the effect of parameter changes and even to incorporate additional features. Starting with McGeer's (McGeer, 1990a) straight-legged model, the addition of knees, articulating feet, lateral motion, and a trunk can be accomplished with little difficulty (Kuo, 1999; McGeer, 1990b; Wisse, 2005).

Stability may be examined qualitatively in terms of collisions and the motion of the swing leg. Relative to the hip, the swing leg swings forward and then backward relative to the COM just before the heelstrike collision. This backward motion causes the length of a particular step to vary if the stance leg is perturbed. A forward perturbation will cause the stance leg to move faster and the collision to occur sooner, resulting in a slightly longer step than normal. The longer step dissipates slightly more energy than usual, because longer steps entail larger changes in the direction of the COM velocity. Because the perturbation adds energy to the COM, the increase in energy dissipated by the collision tends to return the gait to normal. The converse is true for backward perturbations, which subtract energy from the system, but result in collisions that also dissipate less energy. The combined effect is that the collisions automatically adjust for perturbations, attracting the motion back to the nominal gait.

The forward and backward swinging of the leg is critical to stability. The gait described thus far is one of two that arise naturally, differing slightly in the period of the gait cycle (McGeer, 1990a). The longer period gait includes the backward swinging and is passively stable. The shorter period gait, not considered here, does not have a backward swing and is unstable. The heelstrike collision occurs while the swing leg is still moving forward, and might be described as stumbling.

These descriptions are distilled from a more quantitative analysis of limit cycle stability. An entire gait cycle consists of the simulation of one step beginning from a set of initial conditions, ending with the push-off impulse (or alternative means of actuation) and heelstrike collision, and finally accompanied by the appropriate re-labeling of the stance and swing legs (McGeer, 1990a). This operation may be regarded as a single, nonlinear *step-to-step function* taking as input the initial conditions, and yielding as output the initial conditions for a subsequent step. The particular initial conditions that yield a limit cycle are referred to as a *fixed point*, the particular input that repeats itself. Other inputs are generally expected to produce an output that differs from the fixed point. The general effects of small perturbations to the fixed point may be compiled by systematically applying a collection of perturbations spanning all those possible, and examining whether the following steps are closer to the fixed point than the initial, perturbed initial condition. The eigenvalues of the linearized step-to-step function, found via numerical computation, describe the magnification factors of the various perturbations. If all magnifying factors have magnitude less than one, small perturbations are guaranteed to become smaller with each step. This is more precisely termed *local asymptotic stability*, which means that stability only applies to relatively small perturbations, and that the return to the limit cycle may occur in successively smaller steps. Larger perturbations often, but do not necessarily, lead to falling.

Local stability, despite being restricted to small perturbations, is an important feature for walking. The range of perturbations for which local stability applies, known as the *basin of attraction*, is quite small compared to that of humans with central nervous system control. The basin is, however, large enough to be practically useful. Walking robots designed from dynamic walking principles exhibit sufficient practical stability to perform in the real world (Collins et al., 2005). This stability is often not particularly robust, and active feedback control would typically be needed to provide the ability to withstand large perturbations. The far better robustness of human walking may be attributed to a sophisticated control system that can respond to a great variety of disturbances. But even for humans, local stability is helpful because it implies that little control effort need be devoted to the nominal gait, and attention can be devoted to other tasks or to controlling other, less stable components of gait.

Passive stability, even with limited robustness, also has advantages over the more conventional method of controlling the joint trajectories with position control (Kuo, 2007). Joint motions or muscle inputs may be prescribed as functions of time in more complex models, and these may even be designed or optimized to produce a periodic gait. However, these same trajectories do not generally yield stable gaits. In nearly all cases, an arbitrarily small perturbation will result in a fall within one or a few steps. The reason is that prescribed trajectories are inherently feedforward in nature, and do not respond to perturbations. Passive stability is dependent on the passive feedback inherent in the dynamics of a mechanical system, and prescribed motions usually detract from or even eliminate this feedback. More complex models are unlikely to walk stably unless controlled by feedback-generated commands, which themselves are difficult to design.

Dynamic walking stability is also amenable to experimental testing. Passive stability implies little need for feedback control, especially under normal steady walking conditions. A three-dimensional walking model, with the ability to rotate in the frontal plane, displays the same passive stability of the planar models, but with an instability in the lateral direction (Kuo, 1999). The instability can easily be controlled, for example with active feedback control of lateral foot placement. However, this feedback would be expected to be sensitive to imperfections in sensing and control. The result would be greater step variability in the lateral rather than fore-aft directions. In addition, removal of sensory information would be expected to adversely affect lateral step variability rather than fore-aft variability. Measurements of overground step variability in adult subjects ($N = 15$) support both expectations (Bauby & Kuo, 2000), with 79% greater variability in the lateral vs. fore-aft direction. In addition, walking with eyes closed caused disparate increases in variability (53% for lateral direction vs. 21% for fore-aft). Other tests show that artificial enhancement of lateral stability, through elastic cords pulling to the sides during treadmill walking, results in decreased lateral step variability (Donelan et al., 2004), and increased lateral step variability in older adults (Dean, Alexander, & Kuo, 2007), for whom active control of foot placement would be expected to depend on less precise sensing and control. Humans appear to take advantage of passive dynamic stability and perform less control of foot placement in the fore-aft direction compared to the (passively unstable) lateral direction.

This stability is relevant to the present focus on the mechanics and energetics of walking. One reason is that the control of stability appears to require energy expenditure. When subjects are artificially stabilized, they exhibited a decrease in MR of about 9.2% for the same step width (Donelan et al., 2004). The cost appears to be due to the force and work needed to make small adjustments in lateral foot placement, and the increased step-to-step transition costs of more varied foot placements. Passive dynamic stability therefore appears to impact energy expenditure. Another feature of the stabilization results is that they provide an additional avenue for testing dynamic walking models without requiring that they be tailored specifically to the single domain of energetics. A single model simultaneously makes predictions regarding both mechanics and stability, and it is beneficial to test that model in as many domains as possible.

The stability and limit cycle behavior of dynamic walking also contrasts significantly with other approaches. Most other models of walking lack the ability to adjust a periodic gait in response to a parameter value change. For example, models with feedforward joint trajectories are generally only applicable to a single combination of parameters, leading to a fall when applied to a model with slightly altered parameters. Most alterations, for example to the mass distribution of a particular body segment, the shape of the foot, or even to initial conditions, typically do not yield a successful step, much less a periodic or stable gait. One compensation is to use optimization methods to seek a new periodic gait for each altered parameter set. This is an effective means of performing a parameter sensitivity analysis, but one that is computationally costly. An advantage of the dynamic walking is that it is straightforward to determine a limit cycle for a model with perturbed parameters, even if the nominal gait is unstable. Limit cycles are especially easy to solve for with a stable model. The perturbed model need merely be simulated for many steps beginning with the nominal gait's initial conditions, and with each step the model will automatically tend toward its limit cycle. This facilitates the performance of extensive sensitivity analyses (e.g., McGeer, 1990a) that far exceed those performed with other approaches.

4.4. Amount and means of COM velocity redirection predicts step-to-step transition work

In dynamic walking models, the amount of work performed in the step-to-step transition is determined by the velocity change from one step to the next. Reduction of this work would be expected to translate into reduction in energy expenditure. One means by which this work is reduced is by the timing of the human push-off and collision. In the theoretical model, an impulsive push-off immediately preceding the collision will reduce the velocity of collision and decrease the amount of work required. In humans, the push-off does not occur impulsively, but rather over a relatively short amount of time, with each double support accounting for about 10% of a stride. The push-off also begins slightly before the collision, although the two overlap considerably in time (Donelan et al., 2002b). This appears to gain much of the benefit of the theoretical, impulsive push-off, but limited to that which is practical with finite (i. e. imperfectly impulsive) forces.

Another means of reducing step-to-step transition work is to reduce the change in COM velocity dictated by the inverted pendulum. For a given walking speed, little can be done to reduce the magnitude of velocity, but the directional change is subject to modification. McGeer (1990a) showed that arc-shaped feet could improve walking economy by reducing the directional changes in velocity. By rolling on a curved foot, the COM velocity gains a horizontal component proportional to the amount of rolling. For a given speed, the vertical component must undergo less change. The radius of curvature ρ (again normalized by leg length L) determines the step-to-step transition work, approximately proportional to $(1-\rho)^2$ in the simple walking model (see Adamczyk, Collins, & Kuo, 2006). A radius of zero corresponds to a costly compass gait, whereas a radius equal to leg length results in no collision at all. The latter requires unusually long feet, which apparently presents practical issues in implementation. Humans do not have rigid, curved feet, but they actively move the feet and ankles in a manner that resembles a radius of $\rho = 0.3$, in terms of the progression of the center of pressure along the ground (Hansen, Childress, & Knox, 2004; McGeer, 1990a). This theoretically reduces step-to-step transition work by about half compared to compass gait.

We tested the dependence of work and energy expenditure on foot radius of curvature (Adamczyk et al., 2006). We measured adult human subjects ($N = 10$) walking on wooden arcs of varying radii, attached rigidly to the bottoms of stiff boots that prevented ankle motion. The average rate of positive work performed on the COM changed approximately in proportion to $(1-\rho)^2$, as predicted ($R^2 = 0.95$). Work rate decreased by about 58% from a radius increasing from $\rho = 0.02$ to 0.40. For small radii, net metabolic rate also decreased, although with a minimum at $\rho = 0.30$ and increasingly slightly for larger radii. The highest MR was observed for small radii, about 59% greater than the minimum rate. The increasing energetic cost of larger radius arcs appeared to be associated with their length, which created a hyperextension torque about the knee that subjects found to be quite uncomfortable. The knee's structure and the muscles surrounding it appear to be well-optimized for the human foot, and humans do not readily take advantage of increases to the normal effective foot radius and length, even if they reduce step-to-step transition work. The normal effective curvature of about $\rho = 0.30$ nevertheless improves considerably upon a compass gait in terms of both work and energy expenditure.

5. A refined view of the inverted pendulum

The principles of dynamic walking provide insight into what happens between pendulum-like stance phases, and how the swing leg should be controlled. It provides a framework for an entire gait cycle, in which the inverted pendulum remains an apt analogy for the stance leg. However, despite remaining central to the understanding of walking, the inverted pendulum analogy needs refinement. The advantage gained from pendulum-like motion needs to be made more explicit. Because the pendulum-like behavior applies only to the stance phase, the lessons from the dynamic walking approach need to be incorporated to address an entire walking cycle. We therefore propose a refined set of principles that clarify and extend the inverted pendulum analogy.

5.1. The single support leg behaves like an inverted pendulum to transport the COM with relatively little muscle force and work

Dynamic walking models support the inverted pendulum analogy as an important component of economical walking. However, the dynamic walking approach highlights two important advantages of the inverted pendulum. First, dynamic walking models show how the inverted pendulum can perform single support with no work whatsoever. For five decades, the relative advantages and disadvantages of the six determinants and inverted pendulum models have been discussed qualitatively but not with explicit, quantitative models that can be rigorously compared. Dynamic walking models are both specific and quantitative, facilitating direct comparisons with alternative models. Second, the advantage of supporting body weight on a relatively straight leg, resulting in reduced muscle force requirements, has not often been recognized. In dynamic walking, extension of the knee is critical to facilitating a passive dynamic single support phase. In terms of both work and force, the inverted pendulum produces single support more economically than the six determinants model.

5.2. Walking like an inverted pendulum also requires a step-to-step transition, in which work is required to redirect the COM velocity

Although the traditional inverted pendulum analogy describes well one key aspect of walking, it has also neglected an important consequence. The inverted pendulum allows the COM to be transported the distance of a step, following which another step must be commenced. The transition from one step to another, occurring primarily during double support, changes the configuration and ground contact conditions, to produce the initial conditions for another pendulum-like single support phase. This change in configuration necessitates a change in the COM velocity, occurring between the end of one inverted pendulum phase and the beginning of the next. Dynamic walking models, as discussed above, show that the redirection requires work performed by the two legs. Step-to-step transition work may account for 60–70% of the overall metabolic energy expended for walking (Donelan et al., 2002a).

The step-to-step transition resolves a difficulty with the inverted pendulum analogy. As an explanation for energy savings, the pendulum functions all too well, because it actually predicts zero energy expenditure. However, a consequence of an economical, pendulum-like step is the need for a mechanism that enables continuous stepping. The step-to-step transition, and its associated energetic costs, provide both a mechanism and a trade-off that explains and predicts non-zero energy expenditure. Traditional approaches have devoted greater analytical effort to the study of the inverted pendulum, at the expense of the step-to-step transition. The lack of the attention, and perhaps its relatively short duration compared to the entire gait cycle, may have contributed to an assumption that the step-to-step transition was not energetically important. The dynamic walking approach, in contrast, focuses a great deal of attention on this transition and helps to elucidate an important contribution to energy expenditure.

5.3. Forced leg motion can reduce step-to-step transition costs at the expense of energy for force production

The high cost of step-to-step transitions means that it is advantageous to seek walking strategies that can mitigate the cost while maintaining walking speed. Because step-to-step transition work increases sharply with step length, a simple solution is to walk with shorter but faster steps. In dynamic walking models, such steps can be accomplished at no energetic cost by the simple application of a torsional springs acting on the legs about an axis through the hips, speeding leg motion. As discussed above, humans have no such ability to apply spring-like forces passively, but it is nevertheless advantageous to force the legs to move quickly even if the energetic cost is substantial. Isolated leg swinging experiments indicate that this cost is indeed substantial, increasing sharply with frequency. However, compared to the step-to-step transition costs for unforced leg motion, it is advantageous to expend some energy to speed the legs in order to minimize the overall energetic cost. The separate costs of step-to-step transitions and forced leg motions together predict the optimum combination of step length and step frequency for a given speed.

The dynamic walking approach offers a simplified perspective compared to more traditional approaches. By concentrating on the work and forces that produce motion of the legs relative to the body, the dynamic walking approach condenses the complexity of multiple joints and separate stance and swing phases into a single, simple task. This highlights a key area where energetic cost may be due to the production of muscle force rather than only for work.

6. A complementary analogy and new perspectives on walking

If the inverted pendulum analogy applies to single support, there lacks a simple analogy for double support, where the step-to-step transition largely occurs. Here we propose a new analogy to complement the inverted pendulum and to explain redirection of the COM in an intuitive manner. We then use this analogy to consider how the dynamic walking perspective changes how walking should be interpreted.

The proposed analogy is to a ball flying in the air under gravity and periodically redirected between aerial phases by a pair of hands. During the aerial phase, the ball flies in a parabolic arc, with no work or force needed to sustain that motion. The aerial phase requires periodic resetting, where the ball is redirected from its downward and forward motion into an upward and forward motion. One method of performing this redirection would be to allow the ball to bounce, as applies in a typical analogy for running. However, here we specify that the redirection is to be performed by two hands that catch and move the ball through a U-shaped trajectory, with the additional constraint that the ball is very slippery. This means that the hands must produce forces that are perpendicular to the surface of the ball, with the trailing hand producing an upward and forward force, and the leading hand an upward and backward force. A consequence of this task constraint is that, while it is trivial for the hands to perform the redirection, they also perform simultaneous positive and negative work on the ball, with the net work summing to zero. The trailing hand performs positive work, and the leading hand negative work. This may be observed by examining the directions of each hand's force and the ball's velocity. The vector dot product of force with velocity yields the rate of work performed on the ball, a quantity which is positive for one hand and negative for the other.

The analogy to walking is as follows. The inverted pendulum single support phase is similar to the ball's flight phase, in that both are mechanically energy-conservative, requiring neither work nor force to allow the COM or ball to move from one side of the arc to the other. Both also require a redirection in order to commence the next arc. In the case of walking, the two legs perform the redirection. A task constraint requires that the leg forces be directed along the legs toward the COM, there being little ability to generate torque about the center of pressure of ground contact. The analogy helps to illustrate that simultaneous positive and negative work will be performed, and that no net work need be performed in the redirection.

The analogy also helps to illustrate the dynamic walking interpretation of the step-to-step transition as a redirection of COM velocity. In the flying ball task, it would be unusual to interpret the redirection as requiring that the ball be lifted, propelled, or horizontally accelerated. It is easily understood that, at the end of one flight phase, the ball has all of the energy and momentum needed for the next flight phase, but with a velocity that is pointing downward rather than upward. The constraint that positive and negative work will ultimately cancel is due to the directions of the hand forces and the fact that each hand is powered by a separate arm. There is no ability for negative work at one arm to power the positive work at the other. The performance of work would then be expected to expend metabolic energy. This analogy helps to illustrate the goal of redirecting the COM velocity, without need to lift, propel, or accelerate the COM.

It is instructive to also consider other, albeit impractical, means of performing step-to-step transitions at low cost. We note first that the COM can be redirected by one leg during the single support phase with no work. If the inverted pendulum can redirect the COM velocity from upward-and-forward to downward-and-forward with no mechanical work and no energy cost, one may ask why the step-to-step redirection requires work, and whether there is an alternative that costs no energy. The explanation is that the inverted pendulum acts as a workless constraint. The rigid pendulum constrains all of its constituent particles without performing work, and all of the internal forces are perpendicular to the individual particle velocities. Another example of a workless constraint is a smooth, U-shaped wire that redirects the motion of a sliding bead. The wire produces a smoothly varying continuum of reaction forces that are always directed perpendicular to the bead's motion. During double support, the two legs can also redirect the COM velocity in a U shape, but each leg can produce force in essentially one direction along the leg, instead of a smoothly varying continuum as with the U-shaped wire. The resultant force of both legs can redirect the COM while performing zero net work, but individually, each leg performs still work. It is the work of the individual legs that necessarily costs energy.

Another theoretical means of redirecting the COM would be to use the negative work of one leg to power the positive work of the other. An ideal generator could be attached to the leading leg, and used to convert the negative work into energy through regenerative braking. This energy could then be stored in a battery or otherwise transferred to a motor powering the push-off of the trailing leg. The work of the respective legs would then be performed with no metabolic energy cost. Unfortunately, practical issues limit the efficacy of this scenario: real-world implementations of regenerative braking are typically very limited in efficiency, rarely exceeding 25%. There is also little applicability to human walking, because muscles cannot regenerate energy from negative work. At best, negative work can be performed by the body at zero metabolic cost through passive tissue deformation. More realistically, negative work performed actively by muscle costs positive metabolic energy and does not regenerate metabolic fuel.

The example of the bead sliding along a wire does, however, present another theoretical means of improving upon the human step-to-step transition. Here we replace the bead with the human, and imagine a person walking while straddling a discontinuous series of frictionless, U-shaped rails, with the rail sections arranged to coincide with each step-to-step transition. Each single support could be performed normally, with the stance leg supporting body weight and directing the COM along a pendular arc. At the end of each arc, the person would smoothly come into sliding contact with the U-shaped rail, which would gradually receive body weight and then redirect the COM into the beginning of the next step's arc. Comfort issues aside, the redirection could hypothetically be accomplished passively and with no metabolic energy cost.

Workless constraints are not the only means of redirecting the COM at no energy cost. Mechanical springs, for example, can perform positive and negative work while conserving mechanical energy and saving the muscles the effort. A person might walk on a treadmill while suspended from a vertical spring that is slack during single support and taut during double support. The single support phase would occur normally, but the step-to-step redirection could be performed by the spring rather than the legs, with the spring resisting and then reversing the downward motion of the body. Here, negative and then positive work is performed, but unlike the two separate legs, the spring by itself can re-use negative work.

6.1. The COM requires neither propulsion or lifting

Walking has often been interpreted in terms of COM propulsion in the forward direction, or lifting in the upward direction. The propulsion interpretation appears to be rooted in an assumption that walking resembles tasks with an external load, such as pedaling or rowing (Fig. 3a). The lifting interpretation appears to be based on a different assumption, that walking resembles active bobbing of the body up and down (Fig. 3b). In contrast, the dynamic walking perspective is that walking bears greater resemblance to an inter-limb co-contraction task (Fig. 3c and d), as exemplified by the flying ball analogy (Fig. 7).

The propulsion interpretation is superficially appealing, because humans are certainly conscious of the exertion of pushing off during walking. Experience with skating or skiing also seems to apply, so that push-off is interpreted in terms of propelling or horizontally accelerating the COM. The propulsion interpretation is not technically wrong, because push-off does include a component of force in the forward direction. Propulsion is, however, somewhat misleading, because in human walking, the propulsion is not performed against external drag. Instead, the leading leg performs negative work as part of the step-to-step transition. Unlike drag, this negative work is performed purposefully, with equal quantity and importance to the positive work of the trailing leg. As a term, propulsion is

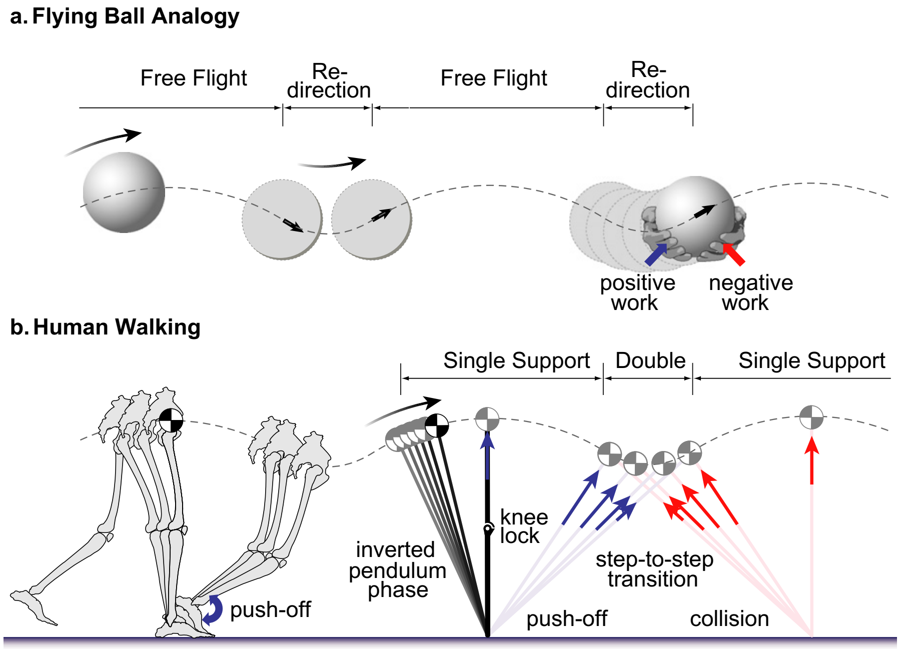

Fig. 7. (a) The flying ball analogy for human walking. A slippery ball flies ballistically in a parabolic arc, and is redirected between free flight phases. The redirection must somehow reverse the downward velocity at the end of one free flight arc, into an upward velocity at the beginning of the next. We imagine that the redirection must be performed by moving two hands that catch the ball, with a fixed relative configuration, with both hands below the ball, and one on each side. Because the ball is slippery, each hand can only apply a force perpendicular to the ball's surface. During the redirection, one hand must perform positive work on the ball, and the other negative work. There is no need to propel, lift, or horizontally accelerate the ball. (b) In the human single support phase, the inverted pendulum supports the COM without requiring work or force, similar to the ball's flight phase. During double support, the COM velocity must be redirected, with each leg's force directed along the leg. The trailing leg performs positive work on the COM, and the leading leg negative work. Although zero net work is performed during double support, the overall effect is to accomplish the redirection required for the next step.

not typically employed to describe a co-contraction task that performs no net work. For example, it would be highly unusual to interpret the task of redirecting the flying ball with two hands in terms of propulsion or drag. To the extent that propulsion is ill-suited to the flying ball task, it is also ill-suited to walking.

A related interpretation attributes the trailing leg with horizontal acceleration of the COM. While it is true that the trailing leg causes the COM to speed up slightly during the step-to-step transition, the gain is immediately negated by the action of the leading leg. The goal of the step-to-step transition is only to change the direction of COM velocity with no need for a net change in speed. In fact, it also possible (though somewhat less economical) to redirect the COM while keeping its forward velocity constant, or even the total velocity magnitude constant (Ruina, Bertram, & Srinivasan, 2005). Discussion of acceleration would be clearer if it were accompanied by equal discussion of deceleration by the other leg. We prefer not to interpret push-off in terms of acceleration, because it implies that an increase in speed is desirable, and because it tends to neglect the importance of the other leg.

Other difficulties appear with the interpretation of lifting the COM. It is true that the trailing leg produces force with an upward component acting on the COM. However, the average vertical force produced by the trailing leg during push-off is actually below body weight, and it is only in combination with the leading leg that the total vertical force exceeds body weight. It is therefore only with both legs that the COM can be lifted, if that interpretation were based on the amount of force. However, even this interpretation is suspect, because over the duration of double support, the net change in COM height is nearly zero. As with the flying ball analogy, the COM ends single support with an appropriate height, momentum, and energy. It requires only redirection and not lifting. The lifting interpretation is apparently based on the vertical bobbing analogy (Fig. 3b). A natural conclusion would be that energy expenditure could be reduced simply by reducing the amount of vertical displacement. The lifting interpretation therefore appears to favor a flattened COM trajectory, just as recommended by the six determinants of gait theory but contradicted by experimental evidence.

It is better to interpret push-off in terms of the net action of the two legs together. The step-to-step transition can produce redirection of COM velocity with no net height change, acceleration, or work. This redirection requires positive and negative work and therefore the expenditure of metabolic energy. We attribute this work to task constraints such as the configuration of the legs and direction of leg forces, rather than to an implied goal of propelling or lifting the body. Of course, a change in the vertical component of COM velocity will necessarily be related to a change in COM height. However, it is the COM velocity rather than height that enters into predictions of step-to-step transition work and energetic cost, and it is the step-to-step transition that differentiates costly changes in velocity from those that occur as a result of pendular motion.

6.2. Work is more relevant than energy exchange

Dynamic walking emphasizes work performed on the COM. Traditional explanations (Cavagna et al., 1963; Donelan et al., 2002b) explain that the inverted pendulum conserves energy by exchanging kinetic and gravitational potential energy. Closer inspection reveals this to be a correct description, but nevertheless a poor explanation. This is because *all* rigid bodies can exchange kinetic and gravitational potential energy when moving. For example, a car rolling up a hill, or any linkage of bodies flying through the air, will also exchange the two types of energy. A person bobbing up and down (Fig. 3b) will not only exchange the two types of mechanical energy, but also require expenditure of considerable metabolic energy. Most relevant is therefore whether work must be performed on a system in order to make it move. Fortunately, the inverted pendulum is indeed one means of conserving mechanical energy without need for work, but it is also not the only means. Mechanical energy may also be conserved without exchanging different types of energy, as in a frictionless car rolling on the level, where kinetic energy is constant. The exchange of different types of mechanical energy is indeed a characteristic, but it is not an explanation of conservation of energy in a pendulum.

The exchange of two types of energy is nevertheless a useful characteristic for evaluating the degree to which human walking resembles a pendulum. Kinetic energy and gravitational potential energy vary out-of-phase during walking, and that offers powerful evidence supporting the pendulum analogy (Cavagna et al., 1963). However, it is also important to apply that analogy only when appropriate, and to consider the possibility that work is performed on a system even when energy is exchanged. For example, it is possible for one muscle to perform positive work, and another an equal amount of negative work on a pendulum. The net rate of work performed on the pendulum would be zero, and kinetic and potential energy would be "exchanged" perfectly. However, from the human's standpoint, the work performed would require expenditure of energy. Double support is one instance in which simultaneous positive and negative work is performed simultaneously (Donelan et al., 2002b), with significant energetic consequences (Donelan et al., 2002a). Exchange of mechanical energy is not sufficient to indicate savings of metabolic energy.

Amount of work provides a better explanation for saving metabolic energy than energy exchange. Humans use the inverted pendulum mechanism to transport the COM during single support with relatively little work. This is clearly advantageous compared to dragging the body over the ground. However, it is also not clearly advantageous compared to flying through the air or gliding on roller skates. The exchange of kinetic and potential energy serves as a useful indication that the pendulum is the appropriate analogy for human single support. The dynamic walking approach also reveals another advantage that is emphasized less often: The structure and configuration of the human leg allows it to behave like an inverted pendulum with relatively little muscle force. A relatively straight leg can support body weight with very little knee torque (Fig. 7b). Even though a leg could also behave like an inverted pendulum with the knee flexed to a fixed angle, there is a considerable advantage to a straighter leg. Work and force together explain the advantage of the inverted pendulum.

It is important to note, however, that even though work performed on the COM during the step-to-step transition is indicative of energetic cost, it is also not the exclusive cost. As evidenced by isolated leg swinging experiments (Fig. 6), the cyclic production of force also appears to expend substantial energy. The total energetic cost of walking is not well-predicted by the work performed on the leg, and instead appears related to both the amplitude of force and its frequency of production (or inverse of burst duration) (Doke & Kuo, 2007). The energetic costs of work and force production together influence walking.

6.3. *Simple models help to study walking constructively*

The dynamic walking approach has been driven by remarkably simple mathematical and physical models. This differs from more traditional approaches, where the human body is often modeled in a more complex manner, generally with many more degrees of freedom, and where joint and force trajectories are interpreted or described from data. In contrast, the dynamic walking approach has generally relied upon quite simple mathematical models, and upon physical machines that are relatively simple compared to more conventional robots. This simplicity is encouraged by the constructive manner by which walking is approached. Dynamic walking is concerned with the elements, in terms of the dynamics of the body and the work performed on the body, that are necessary to construct a periodic gait. More traditional robotics approaches have developed control strategies that are sufficient to produce gait, but usually with greater complexity than necessary. Dynamic walking principles have emerged from remarkably simple but nonetheless useful models.

One advantage of the constructive approach is that it can be demonstrated physically. Mathematical models of dynamic walking are simple enough that they can be realized faithfully with physical machines. The physical demonstrations then test the feasibility and completeness of the mathematical models. The ability to produce complete and fully periodic gaits is especially important, because even though the simple models are often quite approximate, they can be tested in terms of successful gait. A physical demonstration of a robot based on the inverted pendulum analogy alone might produce a correct single support phase, but would not be expected to produce a complete gait. The lack of a periodic gait might easily be overlooked in a mathematical model, but not in a physical demonstration. Physical walking robots (Collins et al., 2005; McGeer, 1990a) demonstrate how inverted pendulum phases can be linked with step-to-step transitions to produce a complete and periodic gait. They also provide reassurance that the simplifying assumptions of dynamic walking are reasonable.

Constructive models also yield testable predictions that can be applied to human experiments. More traditional approaches have been based on human measurements, from which measures such as joint torques can be derived and interpreted. Although interpretation is an important aspect of scientific inquiry, in the case of human walking many of the interpretations have not been subjected to testing, nor have they produced testable predictions. Dynamic walking models produce complete gaits that vary as parameters such as step length or radius of curvature of the foot are manipulated. This facilitates experimental testing where variables may be controlled to produce changes that may be predicted independently from data.

6.4. Dynamic walking models readily incorporate additional features

Although simple models produce complete gaits, this is not to imply that dynamic walking approach model human gait completely. Simple models have indeed proven useful for elucidating the basic principles of walking, but many important details of human walking have yet to be explained. For example, even though dynamic walking emphasizes the importance of push-off, the actual details of which muscles contribute to push-off, and how tendon interacts with muscle, are only gradually being discovered (e.g., Fukunaga et al., 2001). More complex models will become useful as these details are studied. It is unclear, however, whether the next stage of modeling should incorporate all known details and degrees of freedom of humans, or whether they should build upon the simple models of dynamic walking. We advocate the latter as the most effective means of scientific inquiry.

A technological analogy illustrates the importance of simple models and their ability to complement more complex ones. The design of airplanes is highly sophisticated, with complex partial differential equations relating thousands of variables describing the flow of air, the dynamics and vibrations of wings and other bodies, and the motion of the airplane as a whole under many simultaneous loading forces. Their accuracy is sufficient for virtually an entire modern aircraft to be designed on a computer. It may therefore come as a surprise that the complex equations are also never used together as a comprehensive set. In fact, the flight simulators used to train pilots, and the aircraft's internal computations for estimating its own orientation and motion use a far simpler set of equations. The fundamental dynamics of an airplane require only six equations for the translation and rotation of a single rigid body. These equations represent the effects of lift and drag on rigid body motion, and can be represented more exactly (and even unrecognizably) in a more complex model. Despite – and indeed because of – their simplicity, these equations are used throughout the design process and even to drive highly sophisticated flight simulators. The fundamental equations are useful for describing whether the airplane will climb, descend, turn, or spin. These gross actions are much more difficult to identify from the vast array of variables in a complex model, and are also accurate enough to apply in any context where the aircraft's motion as a whole is to be predicted.

Complex equations are, of course, indispensable to modern aircraft design. For example, complex fluid mechanics equations are used for refining airfoil shapes, a process that previously relied heavily on empirical knowledge and heavy experimentation. However, the most common applications are also local in context, such as a fluid flowing uniformly past an airfoil alone, without regard to the structural dynamics of the rest of the airplane. Even though the accuracy of complex equations is critical to refining airfoil shape, that same accuracy provides negligible benefit to describing gross motion, which is better represented with a simple model. A simple model also provides a more practical framework for understanding flight. Every aircraft designer understands and uses the concepts of lift and drag, even though they are merely condensed representations of behaviors more accurately modeled by the fluid mechanics equations, distilling many variables into a few important outcome variables. Even when complex equations are employed, the simple model remains central to the designer's conceptual understanding of how an aircraft takes flight.

We next compare aircraft modeling with the study of human walking. The complex modeling approach attempts to integrate many disparate elements of muscle, skeleton, and nervous system into a comprehensive whole. This is surely a worthy endeavor, but will be much more useful if there is a simple theory providing a global context. The principles of dynamic walking may provide just such a context, within which the gross motion of the body may be understood. As with aircraft modeling, it may prove more practical and useful to employ complex but local models appropriate for specific issues. For example, the design of hip implants might employ sophisticated and accurate finite element models to predict stresses. Still more complex models might be desirable to predict bone resorption as a consequence of stress shielding. However, these models require only the loading conditions of walking, and not a comprehensive model of the entire body. A complex model of walking that incorporates all known physiology may have as little demand as a comprehensive model of an airplane.

A more immediate challenge for highly complex models is in experimentation. Models to date have been subjected to scant testing relative to the number of parameters they contain. As with any complex model, extensive testing is necessary because the integration of many disparate components can lead to unusual conditions that lie outside the intended domain of a particular component. For example, the Hill muscle model (McMahon, 1984) reproduces behaviors derived from empirical measurements for specific conditions such as isometric or isokinetic loading. However, it does not model many known behaviors of muscle, such as fatigue, force enhancement following stretch, or dynamics of calcium transport (e.g., Woledge et al., 1985). These and other behaviors may be relevant in an integrated model of walking even if the Hill model is not meant to apply to such situations. A complex model is therefore best initially assumed inaccurate, and then tested for those conditions where the inaccuracies are most critical. Such testing should also ideally utilize a model and predictions formed independent of the test data, a particularly difficult challenge for some models, where data drives a simulation. This challenge also suggests that the best applications of complex models will not be for conceptual understanding of issues such as the six determinants of gait, but rather for more local applications such as calculation of stress shielding due to a hip implant without need for a model of the entire body.

This is not to endorse the avoidance of complexity altogether, but rather its careful selection, application, and testing. The simple, point-mass models of dynamic walking are useful for their conceptual simplicity, for elucidation of fundamental principles of walking. However, other dynamic walking models also yield insight from greater complexity. Additional model features incorporated to date include human-like mass distribution, knees, torso, three-dimensional motion, and feet with different arc shapes (Adamczyk et al., 2006; Kuo, 1999; McGeer, 1990a, 1990b). Most of these features have been added to relatively modest dynamic walking models, and have yielded simulations amenable to comparison with human walking, with each feature implying at least one comparison. The challenge of simultaneously incorporating many features (e.g., multiple muscles) in a model is the greatly increased number of comparisons or experiments needed to test the model.

Several new features may be incorporated in dynamic walking models while adding only modest complexity. Articulating ankles would allow the feet, presently fixed to the leg in most dynamic walking models (see Fig. 2), to produce more human-like push-off and collision. The impulsive behavior of the model is energetically favorable, because the step-to-step transition requires the least work if the duration and displacement is minimized. This cannot, however, explain why humans perform the transition over the considerable duration of double support, or about 10% of a stride. One explanation is simply that neither push-off nor collision can realistically be performed perfectly impulsively, because of the limited forces that the body structure can sustain. These forces are also supported at least in part by muscles, and the energetic cost of producing force cyclically (see Fig. 6) may also favor longer rather than shorter durations. The actual duration and displacement of the step-to-step transition may therefore be determined by trade-offs between work and force. The addition of articulating ankles would allow a dynamic walking model to perform the step-to-step transition over a range of durations, facilitating the quantitative study of this potential trade-off.

Directly related to the articulation of the ankles is the concurrent motion of the knees. Present dynamic walking models assume full extension of the knees during the step-to-step transition. This assumption simplifies the analysis, but is also clearly unrealistic, as human knees are slightly flexed during double support. The leading knee flexes during the collision, an action that both increases the amount of step-to-step transition work and the vertical displacement of the COM. One potential benefit may be the same as the force trade-off proposed for the articulating ankle, while another may be that the potential storage and return of elastic energy in knee muscles. The return of energy may occur early in single support, when positive work is performed on the COM. Complementary to this action may be the storage of elastic energy in the Achilles tendon following mid-stance (Fukunaga et al., 2001), which appears to contribute to push-off. We have previously termed these actions ‘rebound’ and ‘pre-load’ in recognition of a partially elastic contribution to work performed between, and yet directly related to, step-to-step transitions (Donelan et al., 2002a).

Many more features can certainly be added to dynamic walking models. It is certainly possible to add many features simultaneously, to yield a single model that can incorporate all features in a single package. While this is certainly a reasonable goal, in most cases it is better to take a gradual approach. Dynamic walking models only provide insight when an observed behavior can be definitely attributed to a proposed feature. A more incremental approach, where new features are added singly or in small numbers, facilitates careful analysis and comparison with humans. Dynamic walking models may thus accumulate many new features and capabilities, eventually yielding a single, more comprehensive model.

7. Reconsidering the six determinants of gait

The six determinants of gait have fared poorly in the analyses above. Experimental evidence shows that at least three of the determinants reduce COM displacement to negligible degree (Gard & Childress, 1997, 1999; Kerrigan et al., 2001). It is in any case disadvantageous to do so: When humans intentionally walk with reduced COM displacement, their metabolic energy expenditure increases by a factor of two or more (Gordon et al., 2003; Ortega & Farley, 2005). As explained by a simple model, a flattened COM trajectory requires more work and greater joint torques than a pendular trajectory. A rigorous quantitative examination shows the six determinants not to be explained by COM displacement. They are not determinants at all, and should perhaps be referred to more accurately (albeit less impressively) as the 'six kinematic features of gait'. There is little doubt that normal gait includes these kinematic features, but the mechanistic explanation for these features remains unresolved.

More plausible explanations for the six kinematic features of gait may ultimately arise from dynamic walking models. For example, the motion of the stance foot and knee – the fourth and fifth kinematic features – during collision and push-off may contribute favorably to the step-to-step transition. As discussed above, the non-impulsive nature of the human step-to-step transition requires more work than the theoretically-preferred impulsive transition, but the high cost of actively producing large forces for short durations may favor the former over the latter. The foot is also controlled to produce an effective radius of curvature of $\rho = 0.30$, which also appears to minimize energy expenditure (Adamczyk et al., 2006). The details and mechanisms of these actions have only been explored superficially to date, but it is possible that the fourth and fifth features of gait are advantageous because they smooth out not the displacement of the COM, but the redirection its velocity undergoes. Whether this yields an energetic advantage remains to be determined, and dynamic walking models may be amenable to examining this possibility.

Reduction of COM velocity redirection, however, cannot fully explain the kinematic features of gait. For example, the sixth feature of gait is lateral displacement of the pelvis, "corrected by the existence of the tibiofemoral angle" (Saunders et al., 1953), referring to the normal valgus angle. Dynamic walking models verify that a narrow step width is indeed helpful for reducing step-to-step transition costs (Kuo, 1999), and that narrow steps are accompanied by a low amplitude of lateral COM displacement. However, an abnormal (varus) tibiofemoral angle does not prevent a narrow step width or low lateral displacement of the pelvis, as long as the hip is able to adduct. The varus tibiofemoral angle is probably explained better in terms of structural mechanics, where the tibia's alignment allows it to experience lower bending moments, and hence to remain slender and therefore of low mass compared to the more proximal femur. The tibiofemoral 'correction' does not, however, reduce pelvis displacement.

Regardless of their underlying explanation, the six kinematic features of gait may have practical utility. A clinician who is familiar with them may be observant of abnormal features, which may aid in the assessment and identification of abnormal gait. And even though COM displacement has only indirect relationship to energy expenditure, the normal kinematic features of gait are likely to contribute positively to walking economy. There are few alterations, short of walking more slowly, that would be expected to reduce energy expenditure below that of normal gait. It is therefore sensible to separate the purported determinants of the gait from the kinematic features. Their foundation – the six determinants – may safely be discarded and replaced with a more mechanistic set of explanations yet to be explicated. The discovery of the actual determinants will in due course be improved by rigorous scientific analysis and testing. But even in the absence of viable explanations, the features should continue to be cataloged and quantified for scientific and clinical application.

8. Conclusion

The dynamic walking approach helps to resolve a conflict between two major theories of human gait. A relatively flat COM trajectory, as favored by the six determinants of gait, requires substantial positive and negative work performed by the two legs on the COM, and large-magnitude knee torque to support body weight. The advantage of the inverted pendulum-like gait is that the COM can be transported a step with very little work or torque. However, a consequence of such a gait is the need to transition between pendulum-like steps. Dynamic walking includes the step-to-step transition in a complete, periodic gait cycle, whose motion is generated predominantly by the passive dynamics of the legs themselves. The trailing and leading legs must perform positive and negative work, respectively, on the COM in order to redirect its velocity between steps. This is a form of inter-limb co-contraction that is ideally kept short in both duration and distance. The work of step-to-step transitions is less than that for a flattened COM trajectory, and predicts well a major component of the metabolic cost of human walking. Just as the inverted pendulum analogy explains how single support may be performed with little mechanical work, a new analogy – the flying ball – helps to explain how the COM requires redirection and why work must be performed. The costs of redirection may be reduced by actively speeding the motion of the legs relative to the body, with the human preferred combination of step length and frequency predicted by the trade-off between the costs of step-to-step transitions and actively forcing the legs.

Dynamic walking also offers a unique perspective, because it is constructive rather than descriptive or interpretive. Dynamic walking is concerned with the facilitation of periodic limit cycles, for example by performing sufficient active work to achieve a desired walking speed. The construction of gait from dynamics alone is helpful because it yields predictions independent of experimental data and therefore readily testable by experiment. Another advantage is that simple walking robots can demonstrate the physical principles and test their real-world applicability. Dynamic walking also simplifies the study of gait, because there is no need to assume a desired kinematic or force trajectory, nor to describe or interpret these trajectories, so long as a limit cycle is produced. Trajectories are viewed as outcomes, rather than determinants, of gait.

Appendix A. Details of simple model calculations

Here we provide the mathematical details for simple models of the six determinants theory and step-to-step transition costs. Both models treat the COM as a point mass located at the hip, and predict the work performed on the COM by the legs. In the six determinants model, the COM is assumed to follow a perfectly flat, horizontal trajectory. In the inverted pendulum model, the COM is assumed to follow a circular arc prescribed by the stance leg. The models predict different amounts of work performed on the COM in order to a complete a step.

A.1. Six determinants model

According to the six determinants of gait theory, we constrain the COM to move purely horizontally. The COM is located relative to the stance foot ground contact point (the trailing foot during double support) by horizontal co-ordinate x and constant vertical height h. Separate models are employed for single and double support. During single support, the ground reaction forces in the horizontal and vertical directions are denoted F_x and F_y, respectively. During double support, the two legs' forces are distinguished by subscripts 1 and 2 for the trailing and leading legs, respectively (e.g., F_{1y} is the vertical force of the trailing leg). Forces and positions are defined as positive in the forward and upward directions. We also define the total step length to be s. All forces are normalized by body weight, and all length measurements are normalized by leg length.

The single support stance leg must flex and extend as the COM translates at constant height. By adjusting the effective leg length (the distance from ground contact to the COM), it is possible to maintain a constant COM height, but with varying horizontal velocity. We denote the distance traveled during single support as s_1, with x changing from $-s_1/2$ to $s_1/2$ over the course of one single support phase.

The work performed per step may be calculated as follows. The vertical force must equal body weight, and the ground reaction force must be directed along the leg. These constraints may be expressed as

$$F_y = 1,$$
$$\frac{F_x}{F_y} = \frac{x}{h}.$$

The forces thus determined, the leg will perform first negative work on the COM, and following mid-stance, positive work. The positive work per step during single support is equal to

$$W_1 = \int_0^{s_1/2} F_x \, \mathrm{d}x = \frac{1}{8h} s_1^2,$$

where the integration bounds are restricted to the displacements following mid-stance. The magnitude of negative work, which is entirely performed prior to midstance, is also equal to W_1. Note that the work does not depend on the horizontal velocity, which decreases at the beginning of single support, and then increases following mid-stance. However, because equal amounts of positive and negative work are performed on the COM, it has the same velocity at the beginning and end of single support.

Both legs produce force and perform work against each other during double support. As with single support, constraints act on the legs' ground reaction forces. First, the total vertical leg force must equal body weight,

$$F_{1y} + F_{2y} = 1.$$

Second, the total horizontal leg forces must produce zero net acceleration over double support. There are many ways in which this may be accomplished, but it is most sensible to produce zero acceleration at each instant in time. Doing so minimizes the positive and negative work performed to accelerate and decelerate the COM. This may be accomplished by equating the forward and backward components of force from the two legs,

$$F_{1x} - F_{2x} = 0.$$

The leg forces must also be directed along each leg. This is because the legs have no mass and because by definition, there is no torque acting about the center of pressure of ground contact. The constraints for the two legs are

$$\frac{F_{1x}}{F_{1y}} = \frac{x}{h}$$

and

$$\frac{F_{2x}}{F_{2y}} = \frac{s-x}{h},$$

where step length s is also equal to the distance between the ground contact points. Combining the above equations yields the leg forces

$$F_{1x} = F_{2x} = \frac{-x^2 + sx}{sh},$$
$$F_{1y} = \frac{s-x}{s},$$
$$F_{2y} = \frac{x}{s}.$$

The work performed during double support may be quantified by integrating over the displacement. As with single support, only the horizontal force components contribute to work. But instead of a single leg performing negative and then positive work, double support is characterized by simultaneous work, with the trailing leg performing positive work and the leading leg negative work. The positive work performed is found by integrating over the displacement of double support, from the end of single support to the beginning of the next single support,

$$W_2 = \int_{s_1/2}^{s-s_1/2} F_{1x}\,\mathrm{d}x = \frac{2s^3 - 3ss_1^2 + s_1^3}{12sh}.$$

The total work per step is the sum of that performed during single and double support,

$$W = W_1 + W_2 = \frac{4s^3 - 3ss_1^2 + 2s_1^3}{24sh}.$$

The mechanical cost of transport is the total work per step (already normalized by body weight) divided by the distance traveled s. This cost is lowest if the COM height h is made as large as possible for a given step length. This may be accomplished by constraining double support to begin and end with one leg straight,

$$h = \sqrt{1 - \left(s - \frac{1}{2}s_1\right)^2}.$$

The cost of transport also depends on the relative proportions of single and double support. Inspection of $W_1$and W_2 reveals that double support is more costly than single support. The most economical flat COM trajectory is therefore achieved with single support dominating the step ($s_1 = s$), and with an instantaneous double support phase. The dimensionless positive work rate is

$$\mathrm{WR} = \frac{s^2 f}{4\sqrt{4 - s^2}},$$

where f is step frequency, normalized by the natural frequency of the leg, $g^{0.5}L^{-0.5}$.

A.2. Step-to-step transition model

In dynamic walking, the only energetic cost occurs in step-to-step transitions. The single support phase, in which the stance leg moves like an inverted pendulum, requires no work. The work performed in an impulsive push-off and collision is determined by the magnitude v of COM velocity at the beginning and end of single support, along with the angular displacement of the velocity 2α through the step-to-step transition. Assuming equal magnitudes of trailing and leading leg work, as are required for steady walking powered by push-off, the positive work first redirects the velocity to a purely horizontal direction. The negative work then redirects the velocity further until it is aligned with that prescribed by the ensuing single support phase. The positive work per step performed by the trailing leg in the simple model (Kuo, 2002) is

$$W_1 = \frac{1}{2}v^2 \tan^2 \alpha \approx \frac{1}{2}v^2 \left(\frac{s}{2}\right)^2 = \frac{1}{8}v^2 s^2 = \frac{1}{8}f^2 s^4.$$

An equal amount of negative work, W_2, is also performed. The dimensionless positive work rate is found by multiplying the work per step by step frequency, yielding

$$\mathrm{WR} = \frac{1}{8}f^3 s^4.$$

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