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Extrinsic Calibration of a Camera with Dual 2D Laser Range Sensors for a Mobile Robot

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***Abstract*— A simple and effective algorithm is Proposed for calibrating the extrinsic parameters among a camera and dual laser range sensors whose traces are invisible by using a specially designed checkerboard. On the basis of an analysis of reference coordinates, range data can be transformed into world coordinate, and then a linear solution can be obtained for the problem. The simulation results confirmed that the proposed algorithm can yield good results as compared with a typical calibration method.**

I. Introduction

The Laser Range Sensor (LRS) has been widely used together with a camera for various robot navigation tasks. A LRS is able to directly provide real-time accurate range measurements in large angular fields at a fixed height above the ground plane while a camera acquires rich information such as color, shape, etc. The combination of a LRS and a camera is to obtain more precise measurement and to simplify information processing by making use of their advantages. [1-3].

To fuse the information obtained by the two kinds of sensors, it is critical to obtain the precise homogeneous transformation between the coordinate systems associated to them. The extrinsic calibration parameters are the position and orientation of the camera relative to the range sensors and the intrinsic parameters, such as the calibration matrix of a camera, affect how the sensor samples the scene.

Even though there has been increasing use of 3D laser range finders, they are still lack of portability and flexibility. Furthermore, the time cost of 3D data acquisition is also very expensive so we focus on the pose estimation of camera w.r.t 2D laser range sensors which is cost-effective while provides flexibility and accuracy for range data acquisition. The use of dual range sensors have two advantages. First we can use two measurements of the same object to eliminate invalid data. Second we can detect hanging obstacle like a car's back-box.

In most applications [4-5], calibration methods make use of the visible position of the laser point or stripe. In this paper we consider an extrinsic calibration of a camera with dual laser range sensors where the laser points are invisible to the camera.

A few works can be found on calibrating a camera with range sensors. Zhang and Pless [6] presented a method for the extrinsic parameters calibration based on the constraints between "views"of planar calibration patterns from a camera and LRF in 2004. Bauermann [7] proposed a "joint extrinsic calibration method"based on the minimization of the Euclidean projection error of scene points in many frames captured at different views points. Wasielewski and Strauss [9] proposed a calibration method mainly based on the constraints of projection of the line on the image and the intersection point of the line with the slice plane of the range finder in the world coordinate system. Li and Liu [8] proposed an algorithm for extrinsic parameters calibration of a camera and a laser range finder using line features. But most of the methods are not easily established.

This paper is concerned with external parameter calibration for a camera with dual 2D laser range sensors for a mobile robot under the assumption that the internal sensor calibration is known. A simple and effective algorithm for calibrating the extrinsic parameters is proposed by using a specially designed checkerboard. First, the coordinates of the points on the ground plane are obtained by using the dual LRSs, then a linear solution is obtained by transformation of reference coordinates. It is simple and convenient to use.

The remainder of this paper is organized as follows. Section II introduces the reference coordinate systems and related problems, and extrinsic calibration approach is given in section III. In section IV,we conclude the paper by giving experimental results showing the feature properties of the proposed algorithm.

II. PROBLEM DEFINITION

A. Reference coordinate systems

The objective of camera calibration is to determine a set of camera parameters that describe the mapping between 3-D references coordinates and 2-D image coordinates. In the vision system of robot, the transformation from physics special coordinates to Pixel coordinates involves five reference coordinate systems: Pixel coordinate system, Retinal coordinate system, Camera coordinate system, World coordinate system and Radar coordinate system.

1) Pixel coordinate system: Digital images collected by camera can be stored in computer in the form of array, and each element (Pixel) in the array has a value that is brightness (grey scale) of it. In the Pixel coordinate system, point P(u, v)'s coordinate represents its row number and column number in the array respectively.

2) Retinal coordinate system: In a retinal coordinate system,the origin o is defined as the intersection of the Image plane and the optical axis. The coordinate of o relative to the pixel coordinate system is (u_0, v_0), the physical size of each

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pixel are dx and dy along the X and Y axes respectively, the transformation between two coordinate systems is presented as follow:

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{dx} & s' & u_0 \\ 0 & \frac{1}{dy} & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} \tag{1}$$

where s' is the skew factor that models non-orthogonal X-Y axes.

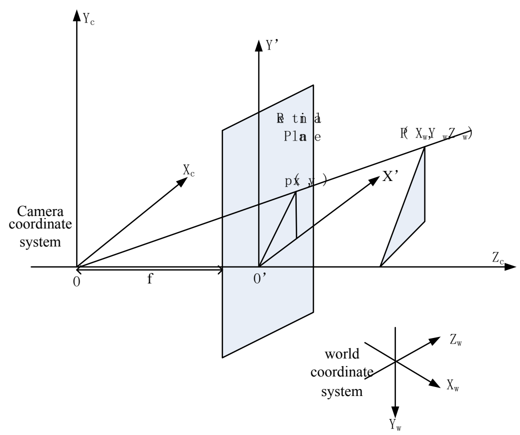


Fig. 1. Camera and World coordinate system

3) Camera coordinate system: Fig.1 illustrates the Camera coordinate system, origin O denotes the camera's optical center point, Camera coordinate system X_c-Y_c axes lays on a plane parallel to the Retinal coordinate system X-Y axes. Z_c axis coincides with the optical axis. The intersection of the optical axis Z_c and the image plane is o$'$. The rectangular coordinate system (X_c Y_c Z_c O) is called camera coordinate system. And the distance oo$'$ is the focal length. The relationship between the Retinal coordinate system and Camera coordinate system is:

$$X = \frac{fX_c}{Z_c}, Y = \frac{fY_c}{Z_c} \tag{2}$$

4) World coordinate system: In real environment, we usually choose a reference coordinate system called world coordinate system to determine the relative position between the object and the camera. And the displacement from the world coordinate system to the camera coordinate system can be described with $\boldsymbol{R}$ (rotation matrix) and $\boldsymbol{t}$ (translation vector). So the homogeneous coordinates of the space point P are (X_w, Y_w ,Z_w $1)^T$ and (X_c, Y_c ,Z_c $1)^T$ respectively, their relationship is shown as follow:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \\ \boldsymbol{0} & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \boldsymbol{H}_1 \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{3}$$

where $\boldsymbol{R}$ is the 3×3 orthogonal identity matrix, $\boldsymbol{t}$ is the three dimensional translation vector, $\boldsymbol{0}$ = $(0,0,0)^T$ and $\boldsymbol{H}_1$ is the association matrix between the two coordinate systems.

5) Radar coordinate system: The raw measurements of LRS are expressed in the polar coordinates on the slice plane as (ρ, θ) , where ρ represents the distance and θ is the rotation angle. The corresponding cartesian coordinate on scanning plane is ($\rho\cos\theta$,$\rho\sin\theta$).

B. Camera model and problem definition

Using the pinhole model of camera and Combining (1),(2)and(3) leads to the following relationship between the Pixel coordinate system and the World coordinate system:

$$Z_c \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} k_x & s & u_0 \\ 0 & k_y & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \boldsymbol{K} \begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \tag{4}$$

Where $k_x = f/dx$, $k_y = f/dy$, $s = s'f$; $[\boldsymbol{R}\ \boldsymbol{t}]$ is entirely determined by the orientation of camera relative to the world coordinate system, which is called the camera' s extrinsic parameters matrix, and it consists the rotation matrix $\boldsymbol{R}$ and the translation vector $\boldsymbol{t}$; $\boldsymbol{K}$ is called the camera intrinsic parameters matrix only related to the camera's internal structure. Here, (u_0, v_0) is called the coordinates of the principal point. kx , ky are the scale factors of image u, v axes respectively. And s is the parameter describing the skewness of the two image axes.

As is clear, given the intrinsic and extrinsic parameters of the camera, to any spatial point, we can calculate the position of the image coordinates (u, v), if we know it's 3D coordinates (X_w, Y_w, Z_w). Let $\boldsymbol{\Phi}$ $(\theta_x, \theta_y, \theta_z)$ be the orientation angles corresponding to the rotation matrix $\boldsymbol{R}$. The objective here is to develop an algorithm for calibrating the orientation vector $\boldsymbol{\Phi}(\theta_x, \theta_y, \theta_z)$ and the translation vector $\boldsymbol{t}(t_x, t_y, t_z)$ of the camera, where the 3D coordinates (X_w, Y_w, Z_w) is obtained by the raw measurements of double 2D range sensor.

III. THE EXTRINSIC CALIBRATION APPROACH

A. Checkerboard Design

Many calibration patterns are used in practice, such as circle, rectangle, chessboard, and triangle and so on. The principle of choosing a pattern is that the feature point or feature line should be easily and accurately detected by both camera and range sensors. In our experiment, a rectangle planner checkerboard was used to simplify the experimentation and to take advantage of each sensor particularity. As shown in Fig. 2, one of the main characteristic of the calibration pattern image is to be composed of two symmetrical zones : a white-like zone and a black-like zone. The symmetrical axes EE$'$ corresponds to the separation of these two zones.

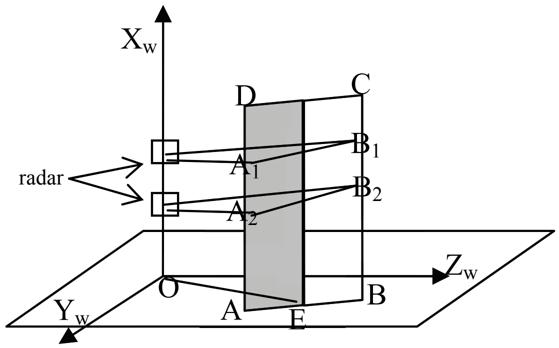


Fig. 2. radar scan plane and checkerboard

B. Data Sampling of 2D Range Sensor

As shown in Fig.2, two 2D LRSs are placed up-down in same direction. A world coordinate system is defined with an origin at the projection of the radar center to the ground plane. OZ_w axis points to the radars' front. OX_w axis is perpendicular to the ground. On the assumption that the ground is a plane, the two laser scan planes are parallel to the plane X=0. We place the checkerboard perpendicular to the ground plane when calibration. Lines A_1B_1 and A_2B_2 are the intersections between scan planes and the checkerboard plane. Point A is projection of points A_1 and A_2 to the ground, and so is point B. Point E is the midpoint of line AB. In one radar coordinate system, coordinates of A_1 and B_1 are (ρ_1 ,θ_1) and (ρ_2 ,θ_2) respectively. And (ρ_3 ,θ_3) and (ρ_4 ,θ_4) for the other LRS. So the world coordinate of the point E(X_w, Y_w, Z_w) on the ground can be derived as:

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} 0 \\ \rho_1 \cos\theta_1 + \rho_2 \cos\theta_2 + \rho_3 \cos\theta_3 + \rho_4 \cos\theta_4 \\ \rho_1 \sin\theta_1 + \rho_2 \sin\theta_2 + \rho_3 \sin\theta_3 + \rho_4 \sin\theta_4 \end{bmatrix} \tag{5}$$

C. Calibration of Extrinsic Parameters

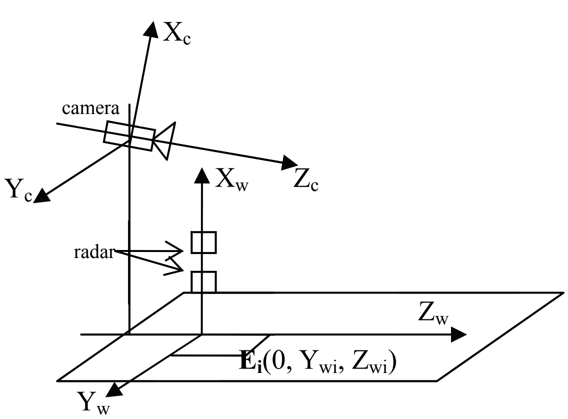


Fig. 3. position and coordinate system

The camera coordinate system is defined as described in II A3) and the world coordinate is defined as described in III B. From formula (5) we obtain n points' coordinates on the ground which denote as $E_i(X_{wi}, Y_{wi}, Z_{wi})$ using LRS measurements. And their corresponding pixel coordinates are $x(u_i, v_i)$. Let's denote the ith column of the rotation matrix $\boldsymbol{R}$ by $\boldsymbol{r_i}$. From formula (4) , we have:

$$\begin{aligned} Z_c \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} &= K[r_1\ r_2\ r_3\ t] \begin{bmatrix} 0 \\ Y_{wi} \\ Z_{wi} \\ 1 \end{bmatrix} \\ &= K[r_2\ r_3\ t] \begin{bmatrix} Y_{wi} \\ Z_{wi} \\ 1 \end{bmatrix} \end{aligned} \tag{6}$$

Therefore, a mapping can be obtained between the ground plane point and its image point:

$$Z_c \boldsymbol{I}_i = \boldsymbol{H}\boldsymbol{P}_i \tag{7}$$

where $\boldsymbol{I}_i = [u_i, v_i, 1]^T, \boldsymbol{P}_i = [y_{wi}, z_{wi}, 1]^T, \boldsymbol{H} = \boldsymbol{K}[\boldsymbol{r_2}\ \boldsymbol{r_3}\ \boldsymbol{t}] = [\boldsymbol{h_1}\ \boldsymbol{h_2}\ \boldsymbol{h_3}]$. From formula (7) we have:

$$\begin{aligned} Z_c u_i &= \boldsymbol{h_1}'^T \boldsymbol{P}_i \\ Z_c v_i &= \boldsymbol{h_2}'^T \boldsymbol{P}_i \\ Z_c &= \boldsymbol{h_3}'^T \boldsymbol{P}_i \end{aligned} \tag{8}$$

By eliminating Z_c, we have:

$$\begin{bmatrix} \boldsymbol{P}_i^T & \boldsymbol{0}^T & -u_i \boldsymbol{P}_i^T \\ \boldsymbol{0}^T & \boldsymbol{P}_i^T & -v_i \boldsymbol{P}_i^T \end{bmatrix} \begin{bmatrix} \boldsymbol{h_1}' \\ \boldsymbol{h_2}' \\ \boldsymbol{h_3}' \end{bmatrix} = \boldsymbol{0} \tag{9}$$

Where $h_i^{'}$ is the transpose of ith row of $\boldsymbol{H}, \boldsymbol{0}^T = [0, 0, 0]$.

When given n points, we have n above equations, which can be written in the form of $\boldsymbol{Ax} = \boldsymbol{0}$, where A is a 2n×9 matrix, $\boldsymbol{x} = [\boldsymbol{h_1}'\ \boldsymbol{h_2}'\ \boldsymbol{h_3}']$ is a 9×1 matrix. The solution is well known to be the right singular vector of A associated with the smallest singular value (or equivalently, the eigenvector of LTL associated with the smallest eigenvalue). And finally we obtain $\boldsymbol{H}$ by $\boldsymbol{x}$.

After $\boldsymbol{H}$ is calculated, we can calculate the extrinsic parameters given the intrinsic matrix $\boldsymbol{K}$:

$$\begin{aligned} \boldsymbol{r_2} &= \lambda \boldsymbol{K}^{-1} \boldsymbol{h_1} \\ \boldsymbol{r_3} &= \lambda \boldsymbol{K}^{-1} \boldsymbol{h_2} \\ \boldsymbol{r_1} &= \boldsymbol{r_2} \times \boldsymbol{r_3} \\ \boldsymbol{t} &= \lambda \boldsymbol{K}^{-1} \boldsymbol{h_3} \end{aligned} \tag{10}$$

Where

$$\lambda = 1/\left\|\boldsymbol{K}^{-1}\ \boldsymbol{h_1}\right\| = 1/\left\|\boldsymbol{K}^{-1}\ \boldsymbol{h_2}\right\|$$

D. Algorithm Summary

The whole algorithm can be described as the following steps:

Step 1: Build a big checkerboard and place it in front of the camera-laser range system in the different orientations.

Step 2: For each checkerboard pose, detect the four intersection points of the laser slice plane with the checkerboard by two LRSs and calculate ground point E's world coordinate by using Eq.(5).

Step 3 Detect the projections of the points Es on the image plane and calculate their coordinates on the image plane.

Step 4: Calculate $\boldsymbol{R}$ and $\boldsymbol{t}$ by (9)(10),and calculate $\boldsymbol{\Phi}$ by $\boldsymbol{R}$[11].

IV. SIMULATION RESULTS

This section illustrates tests on simulated data and demonstrates its performance in Matlab. The performance comparisons between our approach and the conventional approach developed in [8] is carried out to show the advantages of the proposed algorithm.

In the simulation, the true homogenous transformation parameters were defined as the orientation vector $\boldsymbol{\Phi} = [0, -6, 0]^T$ degrees and the translation vector $\boldsymbol{t} = [1.5, 0, 0.25]^T$ meters. In order to make the simulation results more meaningful, we used the real intrinsic matrix $\boldsymbol{K}$ of the camera used in the experiments, which were determined using the calibration tool available at [10]:

$$\boldsymbol{K} = \begin{bmatrix} 787.9576 & 0 & 335.6311 \\ 0 & 788.4153 & 253.2739 \\ 0 & 0 & 1 \end{bmatrix}$$

The relationship between $\boldsymbol{R}$ and $\boldsymbol{\Phi}$ is available in [11].

The size of the checkerboard is 60mm×60mm. The poses of the checkerboard are randomly selected in order to cover the whole image plane.

Li and Liu [8] proposed an algorithm for extrinsic parameters calibration of a camera and a laser range finder using line features. It is the latest and effective work on this topic.

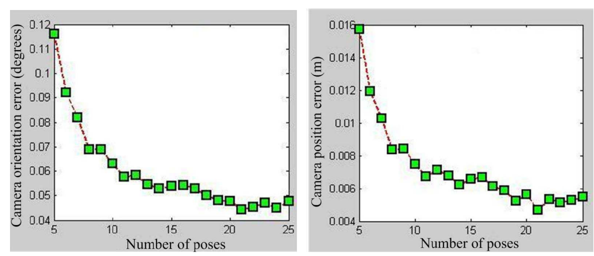
In order to compare with the method proposed by [8], Gaussian noises with mean 0 and standard deviation 0.5 pixels were added to the projected image points. And the 50(mm) uniform noises were also added to the intersection points between the checkerboard and the radar slice plane detected by the double LRSs, which are approximately the same as the observed noise distribution in our sensors. In the simulations, we conducted 100 trials of independent positions of the checkerboard. The average errors of the 100 trails were used as the results. The Gaussian noises were also independent among trials, and so were the uniform noises in the laser ranges measurement.

In the experiment, the estimated extrinsic parameters are compared with the real ones. The camera orientation error $\boldsymbol{\Phi}$ is computed by using the angle between the estimate and the true orientation, and the position error $\boldsymbol{t}$ is computed by using the distance between the estimate and the true camera position. We analyzed calibration errors with different numbers of positions, different orientations and different distances between the double LRS and the checkerboard. In each experiment we compare the result of using one LRS and two LRSs. The parameters and the corresponding results are summarized in the table I.

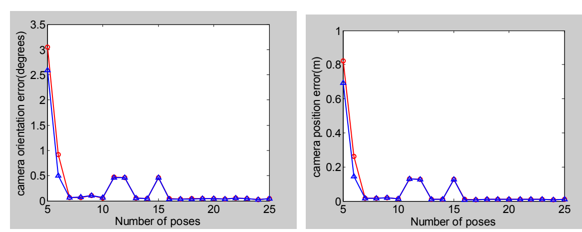
TABLE I
THE PARAMETERS FOR THE PERFORMANCES

Performances		1	2	3
Four elements	Number of the checkerboard	5-25	25	25
	Orientation of the checkerboard	90	60-90	90
	Distance from checkerboard to LRS (m)	10	10	5-25
	Number of Range Sensor	1-2	1-2	1-2
Experimental results		Fig.4	Fig.5	Fig.6

Performance w.r.t. the number of checkerboard poses. This experiment aims to show how the number of plane poses effects the performance. We change the number of poses from 5 to 25. As shown in Fig. 4, the errors decrease with the increase of the number of positions. Comparison between our algorithm and that proposed in [8] led to the similar errors in the orientation when more than 15 positions were used.



(a) The experimental result published in [8]

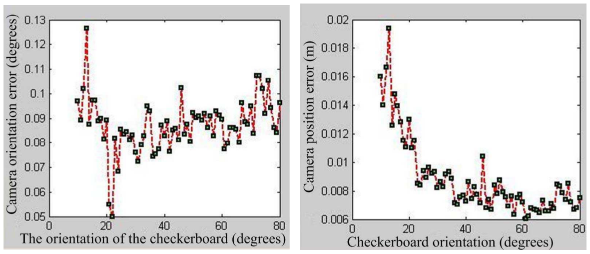


(b) The experimental result of our method

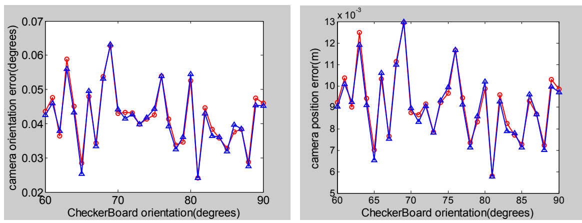
Fig. 4. Errors vs. the number of the checkerboard positions

Performance w.r.t. the orientation of checkerboard plane. This experiment is performed for different orientations of the checkerboard plane to examine its influence. The angles from the checkerboard plane to the radar scan bean varied from 60-90 degrees as shown at the Fig.5. It can be concluded that the calibration errors are hardly affected by those angles in our method while in [8] weekly affected.

Performance w.r.t. the distance between the checkerboard and the LRS. This experiment will show the calibration performance by changing distances between the checkerboard plane and LRS from 5m to 25 m. It is shown in Fig.6 that the calibration performance deteriorated with the increase of the distance because the radar measurements are strongly influenced by distances.



(a) The experimental result published in [8]



(b) The experimental result of our method

Fig. 5. Errors vs. the orientation of the checkerboard

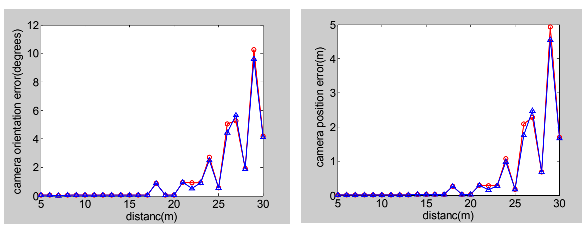


Fig. 6. Errors vs. the distance of the checkerboard

Performance w.r.t. the Number of LRSs. As shown in Fig.4-6, the red line shows the result of one LRS while the blue one for the two LRSs. It is concluded that the calibration performance is weakly influenced by the number of LSRs. Advantages of using two laser sensors are two folds. We can compare two measurements of the same object to eliminate invalid data. Furthermore, hanging obstacle like car's back-box can be detected easily.

V. CONCLUSION

In this paper, a new algorithm has proposed for calibrating the extrinsic parameters of a camera with dual Laser Range Sensors using a newly designed checkerboard. The proposed method requires a few poses of planar pattern which is visible for both the camera and the dual LRSs. This algorithm has the advantages of being simple and convenient to use and yielding good performance to meet many robotic vision tasks.

Because of limited time and space, we did not present experiments with real data and lens distoration was not take into account either, which are to be considered later.

VI. ACKNOWLEDGMENT

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