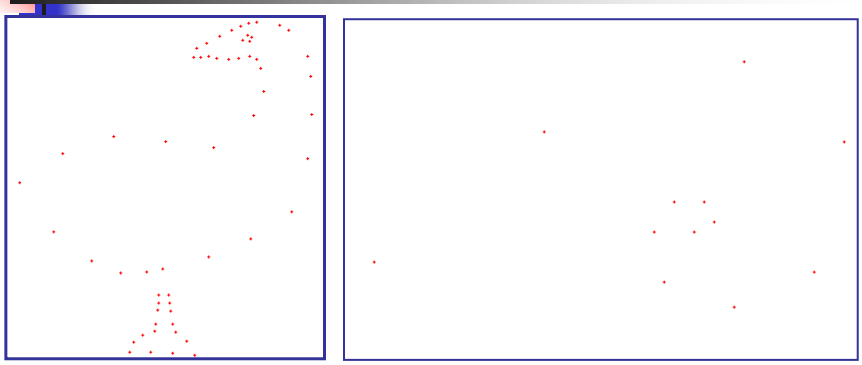
University of
British Columbia

Reconstruction – Crust Techniques

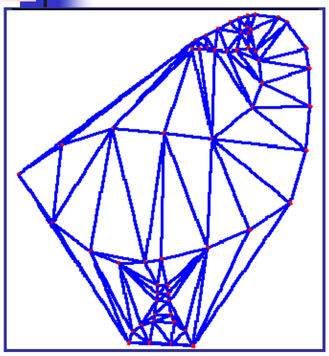
Curve from points- connect the dots

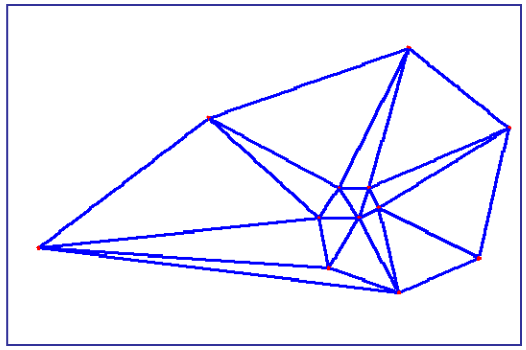


- How to find "correct" connectivity?
- Observation 1: Edges should be far from other points

University of
British Columbia

Curve from points- connect the dots

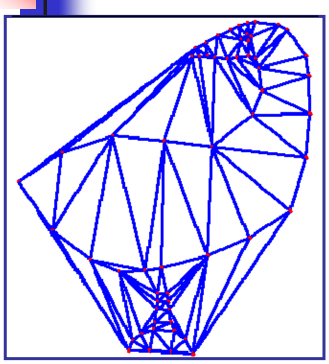


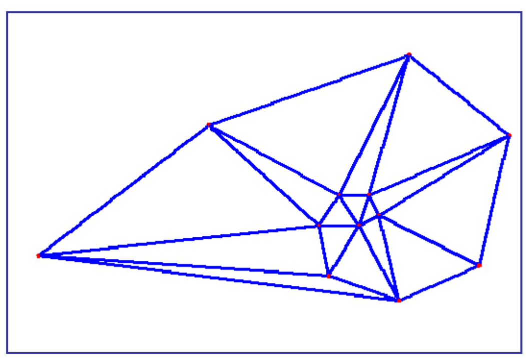


- Provided by Delaunay Triangulation
- Edge e is Delaunay $\Leftrightarrow$ circumcircle of e contains no other sample points

University of British Columbia

Curve from points- connect the dots





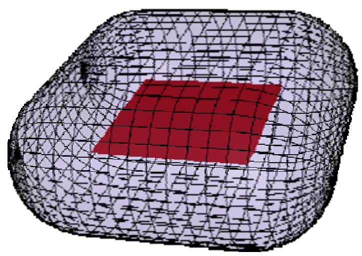
- Not enough (not curve)
- Need shape “structure” information - skeleton

University of British Columbia

Medial Axis &

- ***Medial axis*** of $(d\text{-}1)$-dimensional surface in R^d - set of points with more than one closest point on the surface
- Alternative definition: locus of centers of maximal inscribed spheres

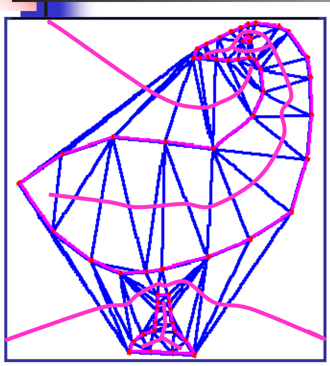
2D example

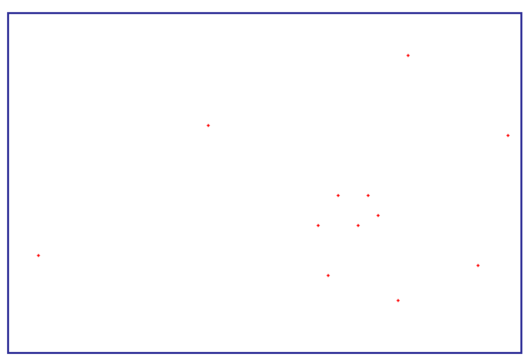


3D example

University of British Columbia

Curve from points- connect the dots

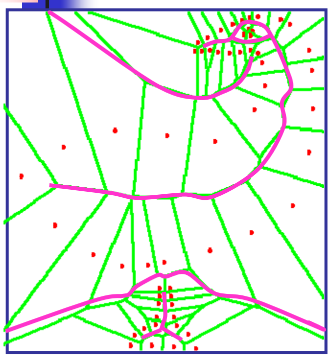


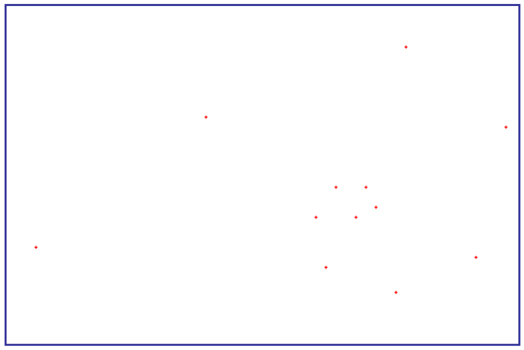


- Connect points with edges which are “far” from curve Medial Axis
- But MA unknown

University of British Columbia

Curve from points- connect the dots



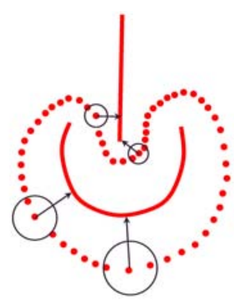


- Voronoi diagram of set of points on curve approximates Medial Axis – if points sampled densely enough

University of British Columbia

Sampling Criterion

- Good sample - sampling density (at least) inversely proportional to distance fro medial axis
- ***r-sample*** : distance from any point on surface to nearest sample point $\leq r$
  - r - distance from point to medial axis
- In general, $r \in (0,1]$
- $r=0.5$ good enough…

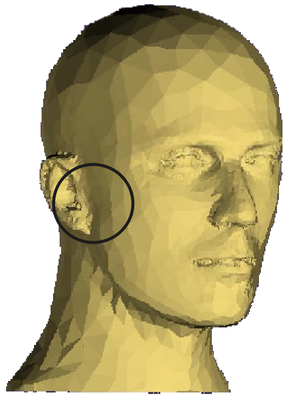


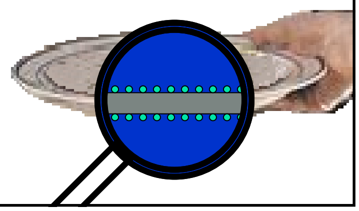
r = 0.5

University of British Columbia

Sampling Criterion (cont.)

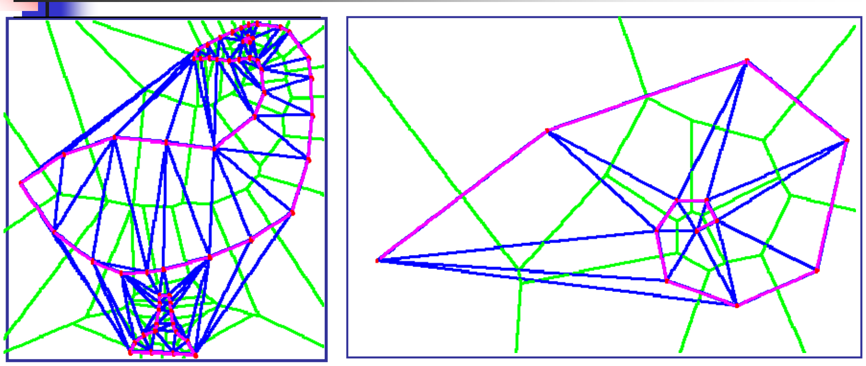
- Inherently takes into account
  - ***curvature*** of the surface
  - ***proximity*** of other parts of the surface





University of British Columbia

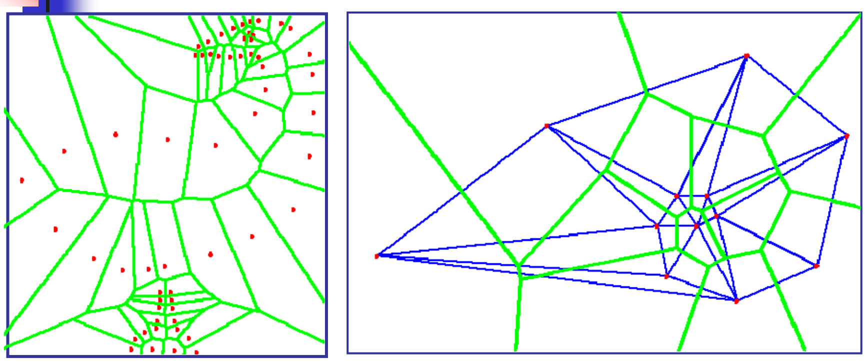
Curve from points- connect the dots



- Use Voronoi vertices to represent MA
- Edge e in **crust** $\Leftrightarrow$ circumcircle of e contains no other sample points or Voronoi vertices of S

University of British Columbia

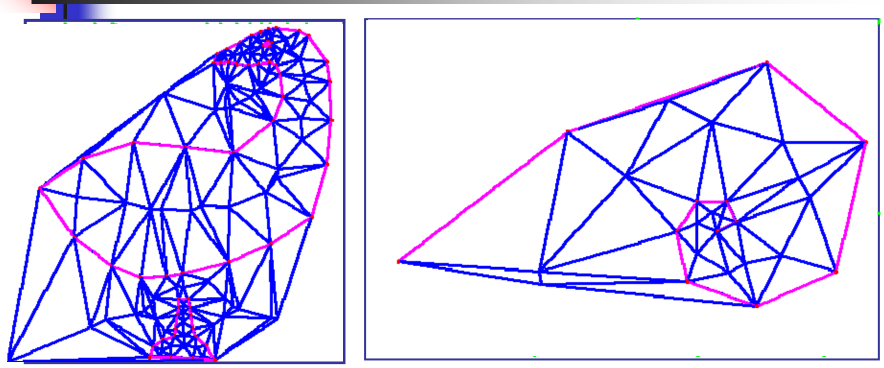
Crust: Algorithm



- Compute Voronoi diagram of S
  - Let V be set of Voronoi vertices

University of British Columbia

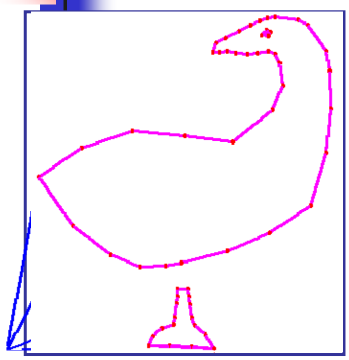
Crust: Algorithm

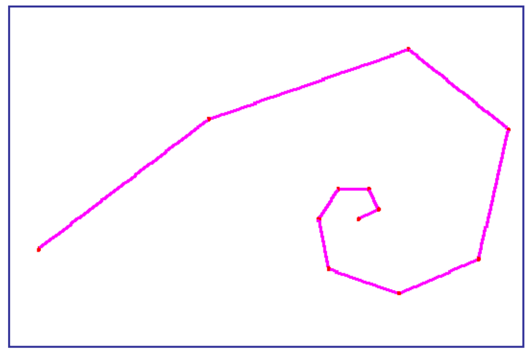


- Compute Voronoi diagram of S
  - Let V be set of Voronoi vertices
- Compute Delaunay Triangulation of $S \cup V$

University of British Columbia

Crust: Algorithm





- Compute Voronoi diagram of S
  - Let V be set of Voronoi vertices
- Compute Delaunay Triangulation of $S \cup V$
- Crust = all edges between points of S

University of British Columbia

Crust Algorithm – Theory

- <u>Theorem 1</u>: The crust of an r-sample from a smooth curve F, for $r \leq 0.25$ connects only adjacent samples of F

Smooth curve F

- The algorithm may fail when r is too big

University of British Columbia

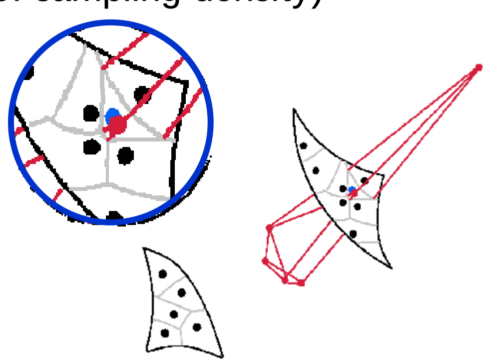
Crust Algorithm – in 2D (cont.)

- Nice Applet:
- http://valis.cs.uiuc.edu/~sariel/research/CG/applets/Crust/Crust.html

University of British Columbia

3D Crust Algorithm

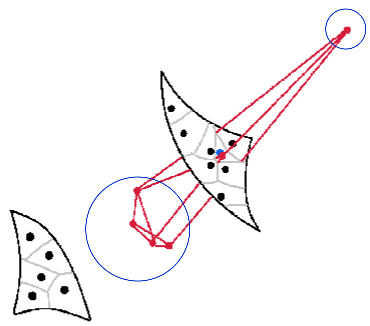
- Extend 2D approach
- Voronoi vertex is equidistant from 4 sample points
- BUT in 3D not all Voronoi vertices are near medial axis (regardless of sampling density)



University of British Columbia

3D Crust Algorithm

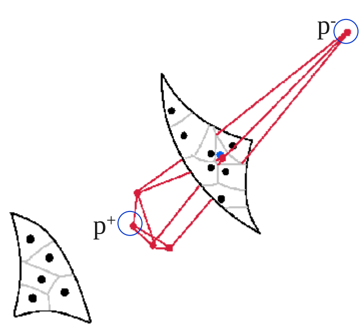
- But **some** vertices of the Voronoi cell are near medial axis
- Intuitively – cell is closed not just from the sides but also from “top” & “bottom”



University of British Columbia

3D Crust Algorithm

- Solution: use only two farthest vertices of V_s - one on each side of the surface
- Call vertices *poles* of s (p^+, p^-)



University of British Columbia

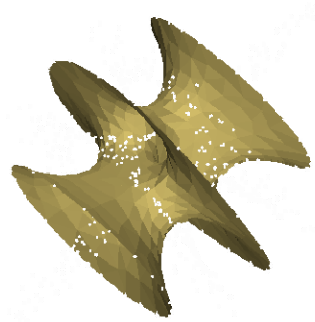
3D Algorithm (basic)

- Compute Voronoi diagram of S
- For each *s* in S find (p^+, p^-)
  - How?
- Let P be the set of all poles p^+ and p^-
- Compute Delaunay triangulation T of S U P
- Add to crust all triangles in T with vertices in S

University of British Columbia

Reconstruction Example

- Crust of set of points and poles used in its reconstruction



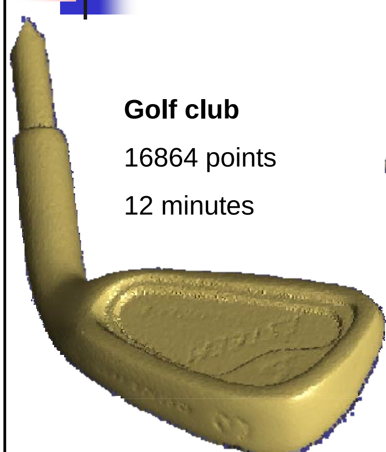
University of British Columbia

Time Complexity

- Time complexity is $O(n^2)$ where $n = |S|$
- Upper bound on 3D Delaunay triangulation
- Almost never arises in practice
- Other steps are linear (but expensive)

University of British Columbia

Reconstruction Examples



Golf club

16864 points

12 minutes

Bunny

35947 points

23 minutes

Foot

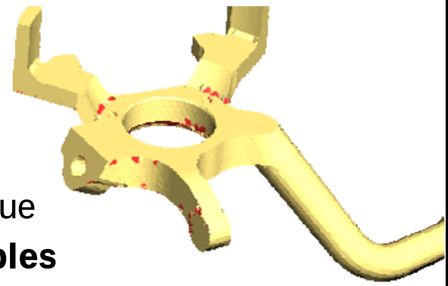
20021 points

15 minutes

University of British Columbia

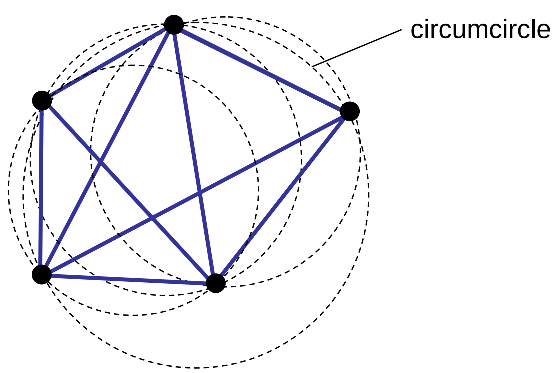
Problems & Modifications

- Alternative pole choice (better reconstruction): farthest & second farthest Voronoi vertices, regardless of direction
- Plus: Correctness
- Minus:
  - Slow –less of an issue
  - **Need dense samples**
  - Problems at sharp corners
  - Noise



University of British Columbia

Delaunay Triangulation



Empty Circle Property:
No other vertex is contained within the circumcircle of any triangle

University of British Columbia

Delaunay Triangulation

- Obeys empty-circle property
- Exists for any set of vertices
- Is **unique** (up to degenerate cases)
- Proven to provide best triangles in terms of quality for given vertex positions
- To test – enough to check pairs of triangles sharing common edge

University of British Columbia