Iterative Closest Point (ICP) Algorithm.

L_1 *solution...*

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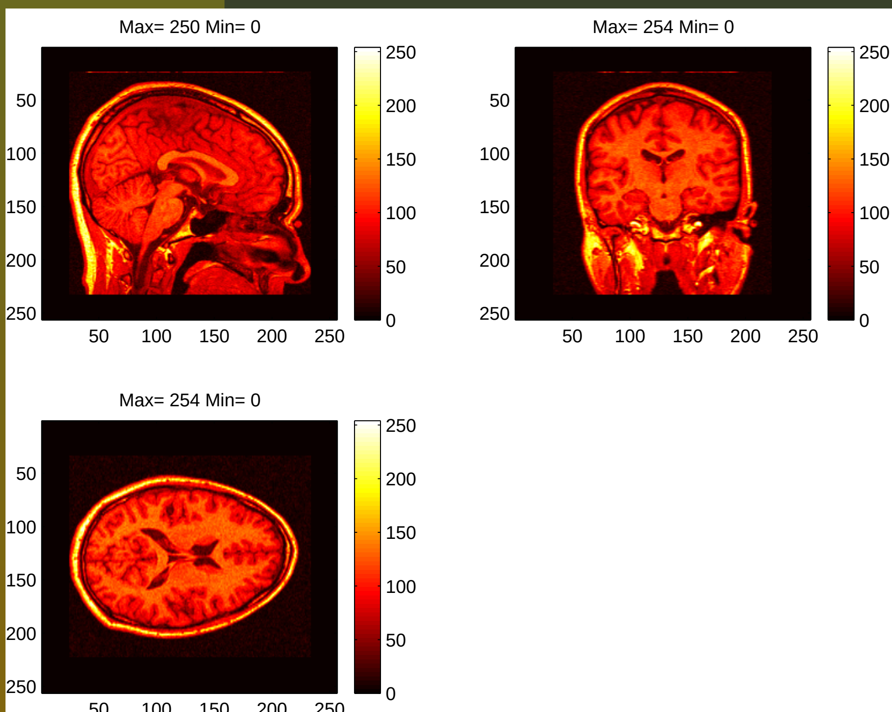
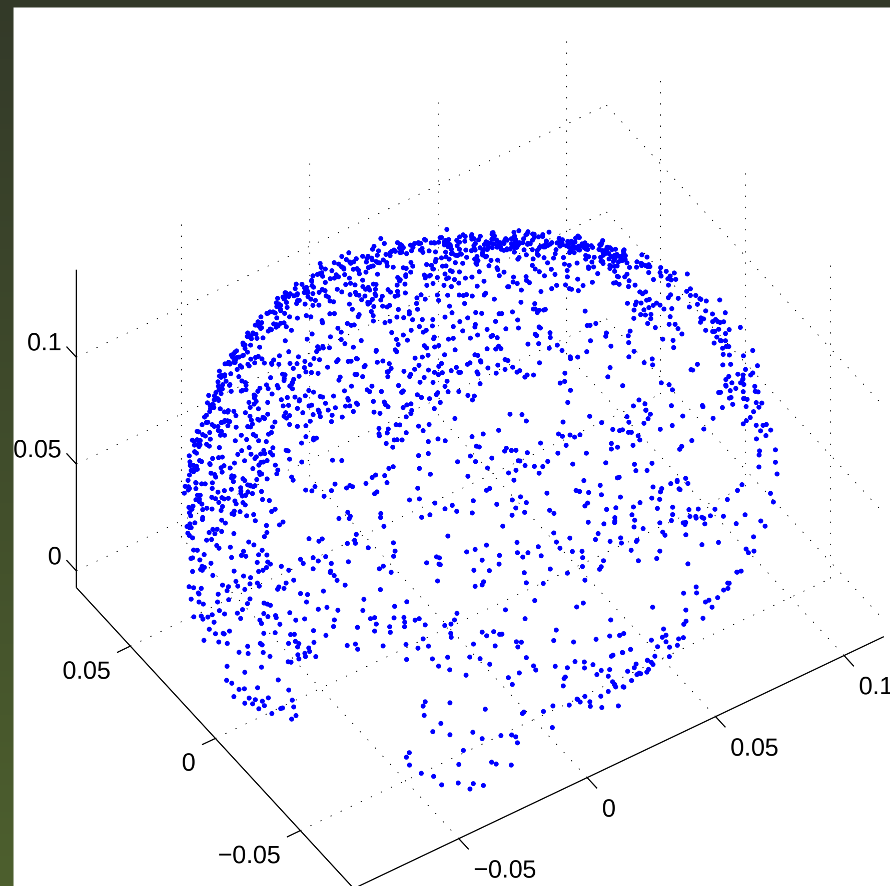
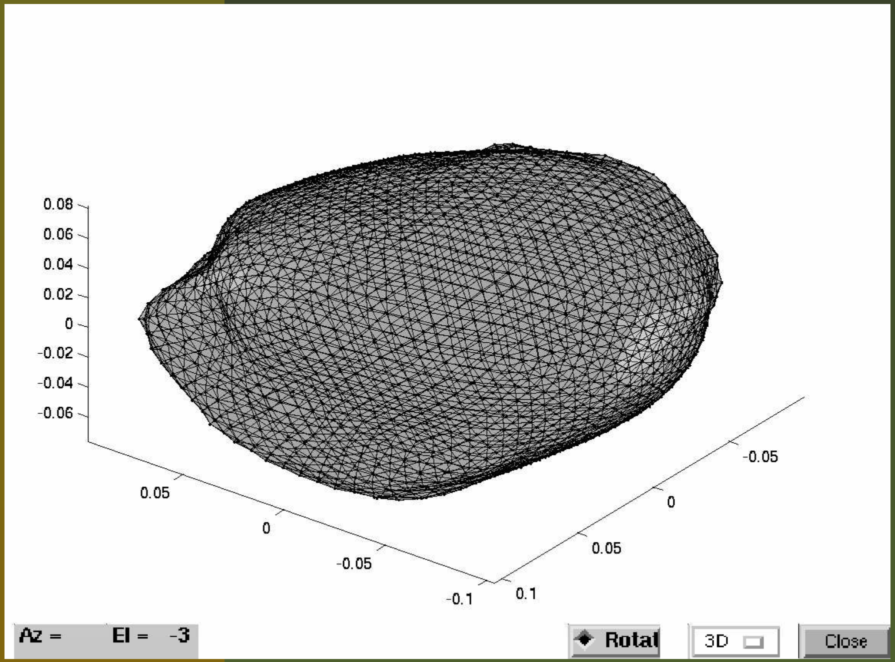
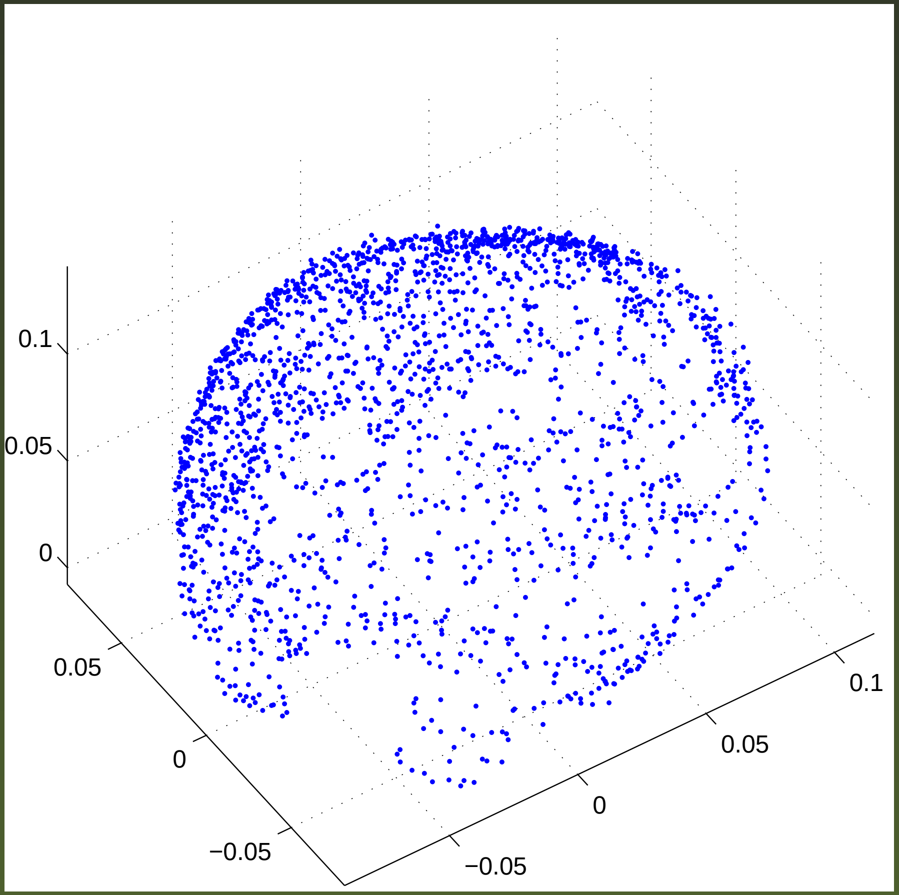
Registration





Registration





Iterative Closest Point

ICP is a straightforward method [Besl 1992] to align two free-form shapes (model X, object P):

- Initial transformation
- Iterative procedure to converge to local minima
  1. $\forall p \in P$ find closest point $x \in X$
  2. Transform $P_{k+1} \leftarrow Q(P_k)$ to minimize distances between each p and x
  3. Terminate when change in the error falls below a preset threshold
- Choose the best among found solutions for different initial positions

Specifics of Original ICP

- Converges to local minima
- Based on minimizing squared-error
- Suggests ‘Accelerated ICP’

ICP Refinements

Different methods/strategies

- to speed-up closest point selection
  - K-d trees, dynamic caching
  - sampling of model and object points
- to avoid local minima
  - removal of outliers
  - stochastic ICP, simulated annealing, weighting
  - use other metrics (point-to-surface vs -point)
  - use additional information besides geometry (color, curvature)

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All closed-form solutions are for squared-error on distances

Found on the Web

- Tons of papers/reviews/articles
- No publicly available Matlab code
- Registration Magic Toolkit (http://asad.ods.org/RegMagicTKDoc) - full featured registration toolkit with modified ICP

Implemented in This Work

- Original ICP Method [Besl 1992]
- Choice for caching of computed distances

Absolute Distances or L_1 norm

Why bother?

- More stable to presence of outliers
- Better statistical estimator in case of non-gaussian noise (sparse, high-kurtosis)
- might help to avoid local minima's

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How?

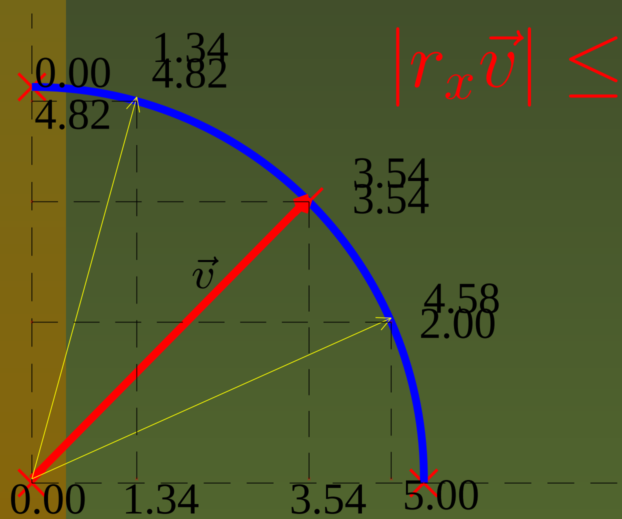
- use some parametric approximation for $y = |x|$ and do non-linear optimization
- present this as a convex linear programming problem

LP: Formulation

Absolute Values $y = |x|$

$x \leq y$ and $-x \leq y$ while minimizing y

Euclidean Distance $\|\vec{v}\| = \sqrt{v_x^2 + v_y^2}$



$|r_x \vec{v}| \leq \|\vec{v}\|, |r_y \vec{v}| \leq \|\vec{v}\|$

LP: Rigid Transformation

Arguments: rotation matrix R and translation vector $\vec{t}$

Rigid Transformation:

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Rigid Transformation:

$$\dot{\vec{p}} = R\vec{p} + \vec{t}$$

Problem: How to ensure that R is rotation matrix?

"Solution": Take a set of "support" vectors in object space and specify their length explicitly.

$$||\dot{\vec{p}}_j - \dot{\vec{p}}_k|| - ||\vec{p}_j - \vec{p}_k|| = 0 \qquad \vec{p}_i, \vec{p}_j \in P$$

LP

$$\vec{\dot{p}} = R\vec{p} + \vec{t}$$

$$\|\vec{\dot{p}}_i - \vec{x}_i\| - d_i = 0 \qquad \forall i, \text{ s.t. } \vec{p}_i \in P, \vec{x}_i \in X$$

$$\|\vec{\dot{p}}_j - \vec{\dot{p}}_k\| - \|\vec{p}_j - \vec{p}_k\| = 0 \qquad \vec{p}_i, \vec{p}_j \in P$$

Objective: minimize $C = \sum_i d_i$

LP: Problems

- Contraction (shrinking):

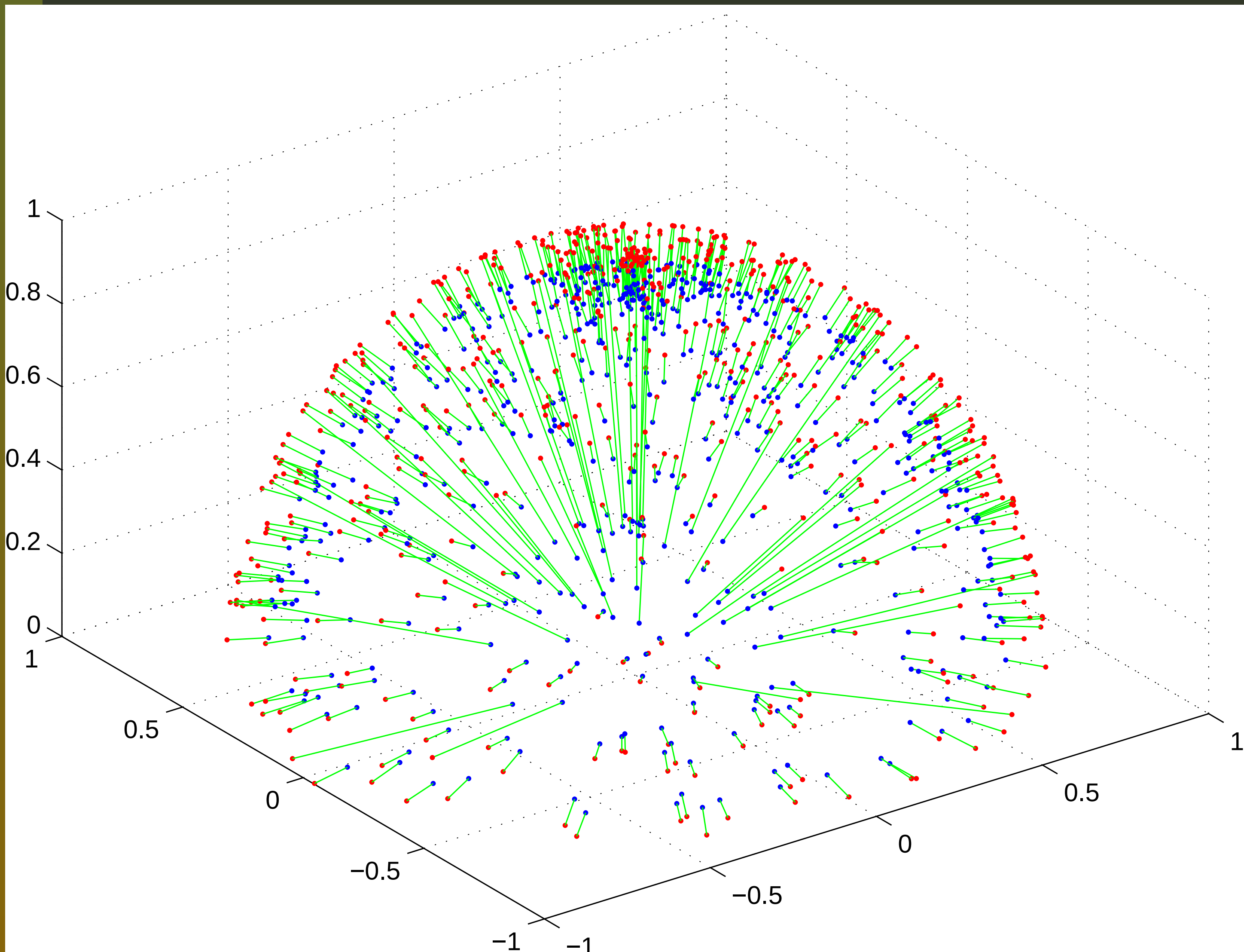
$$\|\vec{\dot{p}}_j - \vec{\dot{p}}_k\| - \|\vec{p}_j - \vec{p}_k\| = 0$$

is actually

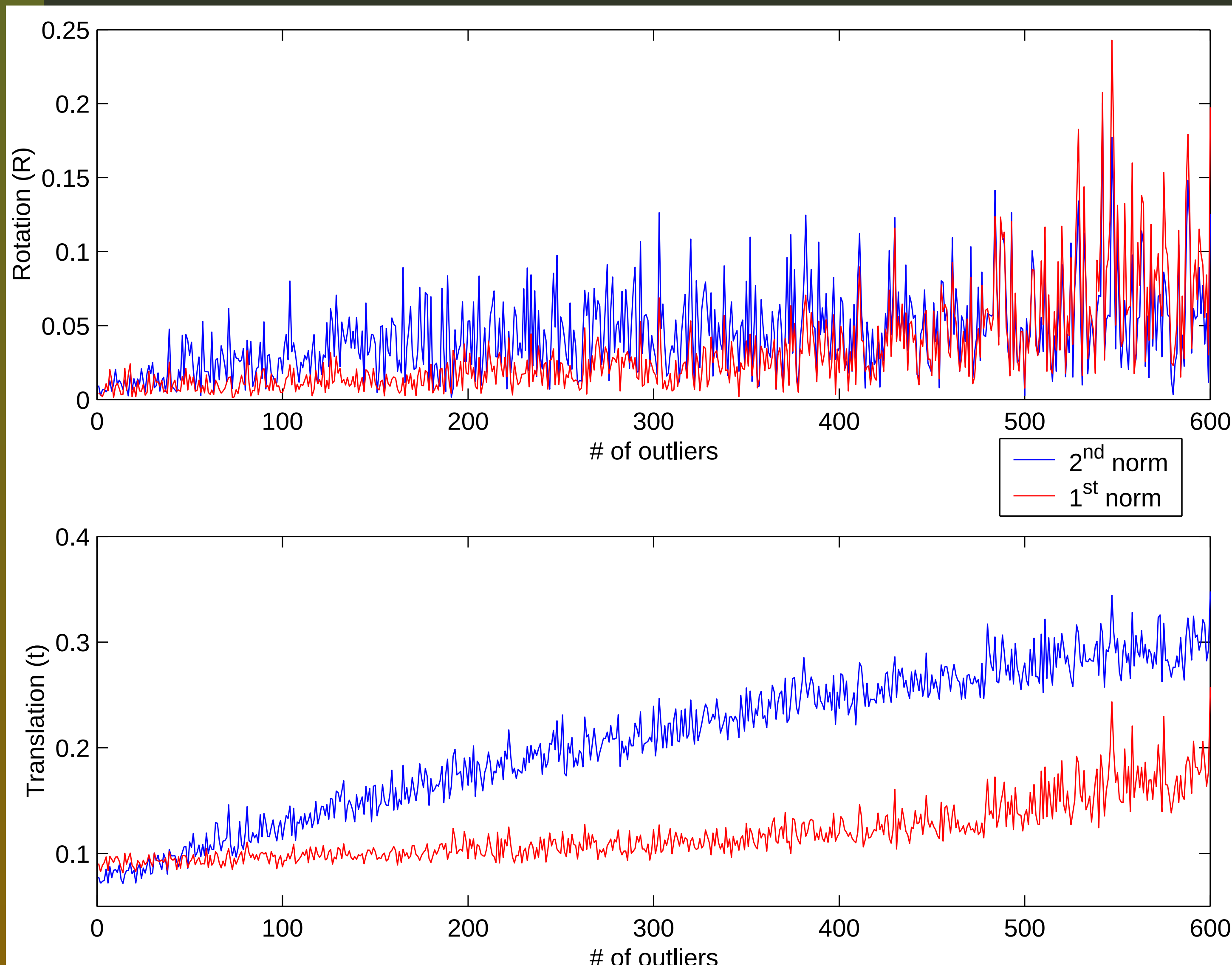
$$\|\vec{\dot{p}}_j - \vec{\dot{p}}_k\| - \|\vec{p}_j - \vec{p}_k\| \leq 0$$

- R matrix needs to be "normalized" to the nearest orthonormal matrix due to our $\|x\|$ LP approximation even if no contraction occurred.

LP: Results



LP: Results


Rotation (R)
0.25
0.2
0.15
0.1
0.05
0
0
100
200
300
400
500
600
# of outliers
2nd norm
1st norm
Translation (t)
0.4
0.3
0.2
0.1
0
100
200
300
400
500
600
# of outliers

LP: Conclusions

- Presented problem is suitable to minimize L_1 error instead of L_2 error commonly used.
- Using L_1 norm improved solution in the presence of strong outliers.