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# Adaptive Stabilization and Trajectory Tracking of Airship with Neutral Buoyancy

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**Abstract** This paper presents an adaptive nonlinear control solution to the horizontal motion of an autonomous airship. We define a novel family of error functions including configuration error and velocity error. Then, we establish the error system. The adaptive nonlinear controller stabilizing the error system is designed by Lyapunov direct method and Matrosov theorem. Numerical studies are presented to illustrate the effectiveness of the proposed controller.

**Key words** Adaptive nonlinear control, Lyapunov method, Matrosov theorem, autonomous airship, neutral buoyancy

In recent years, autonomous airships have become an intense research area all over the world for their emerging applications such as communication platform, surveillance, advertising, monitoring, inspection, exploration, and so on[1]. Many published works on airship control theme appeared in the last decade. Reference [2] introduced a robust output tracking controller for airship′s attitude using Lyapunov method. Reference [3] analyzed the stability and robustness of airships control system using dynamic inversion method. Reference [4] designed a time-varying controller for a nonlinear underactuated autonomous airship using averaging and backstepping approaches. Since the dynamics of autonomous airships is highly nonlinear and the aerodynamic coefficients of airships are difficult to estimate accurately, the controller design is required to have learning and adapting abilities in order to provide desired performance. However, there are few published works about applying adaptive control method to autonomous airships though this method has been known for many years and applied successfully in many other fields[5−7]. In this paper, we present an adaptive control method for autonomous airship with neutral buoyancy. We define a family of configuration errors and velocity errors, and then, establish the error system. By applying Lyapunov direct method, we construct an adaptive control law for the developed error system, and then, prove that the closed-loop error system is locally uniformly asymptotically stable using Matrosov theorem.

The organization of this paper is as follows. A simple model characterizing the planar motion of an airship is given in Section 1. By introducing a novel family of configuration and velocity errors, the model is transformed into an error system. Section 2 is devoted to the adaptive controller design for stabilizing the error system. Simulation results are given in Section 3, and Section 4 is the conclusion.

## 1 Problem formulation

### 1.1 System modeling

Consider an airship with neutral buoyancy, i.e., the buoyancy is equal to the gravity. Since the restored torque caused by the noncoincident centers of gravity and buoyancy can stabilize the pitch and roll motions, a three-degree-of-freedom simplified dynamics could model the horizontal motion. For detailed development of the mathematical model of an airship moving with six-degree-of-freedom, the reader can refer to [8]. For the purpose of studying the planar dynamics of airship, we define the inertia frame with the origin $O_i$ at some point fixed at the earth, and the body frame with the origin $O_b$ at the center of volume (CV) (i.e., the center of buoyancy) as shown in Fig. 1. The $O_bX_b$ axis lies along the symmetry axis of airship′s envelope. The $O_bY_b$ axis points to the starboard and is perpendicular to $O_bX_b$.

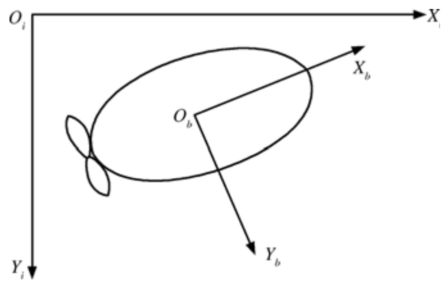


Fig. 1 Inertia frame and body frame

The kinematics of horizontal motion of airship can be described as

$$\dot{\boldsymbol{\zeta}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix} \boldsymbol{\xi} \tag{1}$$

where $\boldsymbol{\zeta} = \begin{bmatrix} \psi & x & y \end{bmatrix}^{\mathrm{T}}$ is the configuration vector, $\boldsymbol{\xi} = \begin{bmatrix} r & u & v \end{bmatrix}^{\mathrm{T}}$ is the velocity vector, $\psi$ is the airship′s yaw angle, $\begin{bmatrix} x, & y \end{bmatrix}$ is the position in inertia frame, $r$ is the yaw angular rate, and $u$ and $v$ are the linear velocities along $O_bX_b$ and $O_bY_b$, respectively. Analyzing the forces and moments endured by airship and using Newton motion law, we get the following dynamics equation of airship (neglecting its longitudinal motion)[9]

$$M_A\dot{\boldsymbol{\xi}} = \overbrace{\begin{bmatrix} 0 & v & -u \\ 0 & 0 & r \\ 0 & -r & 0 \end{bmatrix}}^{G} M_A\boldsymbol{\xi} - D_{diss}\boldsymbol{\xi} + \boldsymbol{\tau} \tag{2}$$

where $M_A = \mathrm{diag}\{J_{33}, m_{11}, m_{22}\}$, $J_{33}$ is the sum of moment of inertia and added moment of inertia, and $m_{11}$ and $m_{22}$ are the sum of airship′s mass and added mass in the directions of $O_bX_b$ and $O_bY_b$, respectively; $D_{diss} = \mathrm{diag}\{d_r, d_u, d_v\}$, $d_r$, $d_u$, and $d_v$ are dissipation coefficients, respectively; $\boldsymbol{\tau} = \begin{bmatrix} \tau_r & \tau_u & \tau_v \end{bmatrix}^{\mathrm{T}}$, $\tau_r$, $\tau_u$, and $\tau_v$ are external control forces and moments, respectively.

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### 1.2 Error definitions

Let $\boldsymbol{\zeta}_d = [\ \psi_d \quad x_d \quad y_d\ ]^{\mathrm{T}}$ denote airship's desired trajectory and $\boldsymbol{\xi}_d = [\ r_d \quad u_d \quad v_d\ ]^{\mathrm{T}}$ denote desired velocity. $\boldsymbol{\zeta}_d$ and $\boldsymbol{\xi}_d$ are related by the following equation named desired trajectory generator

$$\dot{\boldsymbol{\zeta}}_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_d & -\sin\psi_d \\ 0 & \sin\psi_d & \cos\psi_d \end{bmatrix} \boldsymbol{\xi}_d \tag{3}$$

**Definition 1.** Define configuration error $\boldsymbol{\zeta}_e$ and velocity error $\boldsymbol{\xi}_e$ as

$$\boldsymbol{\zeta}_e = \begin{bmatrix} \psi_e \\ x_e \\ y_e \end{bmatrix} = \begin{bmatrix} \psi - \psi_d \\ (x - x_d)\cos\psi_d + (y - y_d)\sin\psi_d \\ -(x - x_d)\sin\psi_d + (y - y_d)\cos\psi_d \end{bmatrix} \tag{4}$$

$$\boldsymbol{\xi}_e = [\ r_e \quad u_e \quad v_e\ ]^{\mathrm{T}} =$$
$$\boldsymbol{\xi} - \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ x_e\sin\psi_e - y_e\cos\psi_e & \cos\psi_e & \sin\psi_e \\ x_e\cos\psi_e + y_e\sin\psi_e & -\sin\psi_e & \cos\psi_e \end{bmatrix}}^{F} \boldsymbol{\xi}_d \tag{5}$$

The configuration error $\boldsymbol{\zeta}_e$ increases when $\|\boldsymbol{\zeta} - \boldsymbol{\zeta}_d\|$ increases and vice versa. The velocity error $\boldsymbol{\xi}_e$ increases with the increase of $\|\boldsymbol{\xi} - \boldsymbol{\xi}_d\|$ and vice versa. The errors are zeroes when $\boldsymbol{\zeta} = \boldsymbol{\zeta}_d$ and $\boldsymbol{\xi} = \boldsymbol{\xi}_d$ hold. So, we can say that the error definitions (4) and (5) are reasonable.

### 1.3 Kinematics and dynamics equations of error system

According to (1) ~ (5), the error kinematics and dynamics equations are developed in this section.

The error kinematics is obtained by differentiating (4):

$$\dot{\boldsymbol{\zeta}}_e = \begin{bmatrix} \dot{\psi}_e \\ \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} r_e \\ \Im \\ \aleph \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_e & -\sin\psi_e \\ 0 & \sin\psi_e & \cos\psi_e \end{bmatrix}}^{A_e} \boldsymbol{\xi}_e \tag{6}$$

where $\Im = u\cos\psi_e - v\sin\psi_e - u_d - r_d[(x - x_d)\sin\psi_d - (y - y_d)\cos\psi_d]$ and $\aleph = u\sin\psi_e + v\cos\psi_e - v_d - r_d[(x - x_d)\cos\psi_d + (y - y_d)\sin\psi_d]$. Differentiating (5), we have the error dynamics

$$\begin{aligned} \dot{\boldsymbol{\xi}}_e = {} & \dot{\boldsymbol{\xi}} - \dot{F}\boldsymbol{\xi}_d - F\dot{\boldsymbol{\xi}}_d = \\ & \dot{\boldsymbol{\xi}} + \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ v_e & 0 & -r_e \\ -u_e & r_e & 0 \end{bmatrix}}^{G_e^{\mathrm{T}}} F\boldsymbol{\xi}_d - F\dot{\boldsymbol{\xi}}_d = \\ & M_A^{-1}GM_A(\boldsymbol{\xi}_e + F\boldsymbol{\xi}_d) - M_A^{-1}D_{diss}(\boldsymbol{\xi}_e + F\boldsymbol{\xi}_d) + \\ & G_e^{\mathrm{T}}F\boldsymbol{\xi}_d - F\dot{\boldsymbol{\xi}}_d + M_A^{-1}\boldsymbol{\tau} \end{aligned} \tag{7}$$

Equations (6) and (7) describe the error system of airship's lateral motion. The objective of control design is to construct control law stabilizing the error system. If the desired velocity $\boldsymbol{\xi}_d = 0$, the airship will be hovering at some point. If the desired velocity $\boldsymbol{\xi}_d \neq 0$, the airship will move along the desired trajectory generated by (3). So the error system brings the positioning control problem and the trajectory tracking problem into an unified framework.

It is easy to verify that the following equations hold

$$G^{\mathrm{T}}\boldsymbol{\xi} = 0 \tag{8}$$

$$G_e^{\mathrm{T}}\boldsymbol{\xi}_e = 0 \tag{9}$$

$$G^{\mathrm{T}}\boldsymbol{\xi}_e + G_e^{\mathrm{T}}\boldsymbol{\xi} = 0 \tag{10}$$

## 2 Adaptive control law design

Denote $\bar{M}_A$ and $\bar{D}_{diss}$ as the estimated values of $M_A$ and $D_{diss}$, respectively. $\tilde{M}_A$ and $\tilde{D}_{diss}$ are the estimation errors, i.e.,

$$\tilde{M}_A = \mathrm{diag}\{\tilde{J}_{33}, \tilde{m}_{11}, \tilde{m}_{22}\} = \bar{M}_A - M_A \tag{11}$$

$$\tilde{D}_{diss} = \mathrm{diag}\{\tilde{d}_r, \tilde{d}_u, \tilde{d}_v\} = \bar{D}_{diss} - D_{diss} \tag{12}$$

Denote $\bar{\boldsymbol{\theta}}_{M_A} = [\bar{J}_{33}, \bar{m}_{11}, \bar{m}_{22}]^{\mathrm{T}}$, $\tilde{\boldsymbol{\theta}}_{M_A} = [\tilde{J}_{33}, \tilde{m}_{11}, \tilde{m}_{22}]^{\mathrm{T}}$, $\bar{\boldsymbol{\theta}}_{D_{diss}} = [\bar{d}_r, \bar{d}_u, \bar{d}_v]^{\mathrm{T}}$, $\tilde{\boldsymbol{\theta}}_{D_{diss}} = [\tilde{d}_r, \tilde{d}_u, \tilde{d}_v]^{\mathrm{T}}$, $\bar{\boldsymbol{\theta}} = [\bar{\boldsymbol{\theta}}_{M_A}, \bar{\boldsymbol{\theta}}_{D_{diss}}]^{\mathrm{T}}$, $\tilde{\boldsymbol{\theta}} = [\tilde{\boldsymbol{\theta}}_{M_A}, \tilde{\boldsymbol{\theta}}_{D_{diss}}]^{\mathrm{T}}$, $\boldsymbol{\theta}_{M_A} = [J_{33}, m_{11}, m_{22}]^{\mathrm{T}}$, $\boldsymbol{\theta}_{D_{diss}} = [d_r, d_u, d_v]^{\mathrm{T}}$, and $\boldsymbol{\theta} = [\boldsymbol{\theta}_{M_A}, \boldsymbol{\theta}_{D_{diss}}]^{\mathrm{T}}$.

The estimation error of parameters $\tilde{\boldsymbol{\theta}}$ is defined by

$$\tilde{\boldsymbol{\theta}} = \bar{\boldsymbol{\theta}} - \boldsymbol{\theta} \tag{13}$$

The operator $Y(\cdot)$ is defined by the following equation. For any $\boldsymbol{x} = [\ x_1 \quad x_2 \quad x_3\ ]^{\mathrm{T}}$,

$$Y(\boldsymbol{x}) = \mathrm{diag}\{x_1, x_2, x_3\} \tag{14}$$

So, we have

$$\begin{aligned} \tilde{M}_A\boldsymbol{x} = {} & \mathrm{diag}\{\tilde{J}_{33}, \tilde{m}_{11}, \tilde{m}_{22}\}[\ x_1 \quad x_2 \quad x_3\ ]^{\mathrm{T}} = \\ & \mathrm{diag}\{x_1, x_2, x_3\}[\ \tilde{J}_{33} \quad \tilde{m}_{11} \quad \tilde{m}_{22}\ ]^{\mathrm{T}} = \\ & Y(\boldsymbol{x})\tilde{\boldsymbol{\theta}}_{M_A} \end{aligned} \tag{15}$$

$$\begin{aligned} \tilde{D}_{diss}\boldsymbol{x} = {} & \mathrm{diag}\{\tilde{d}_r, \tilde{d}_u, \tilde{d}_v\}[\ x_1 \quad x_2 \quad x_3\ ]^{\mathrm{T}} = \\ & \mathrm{diag}\{x_1, x_2, x_3\}[\ \tilde{d}_r \quad \tilde{d}_u \quad \tilde{d}_v\ ]^{\mathrm{T}} = \\ & Y(\boldsymbol{x})\tilde{\boldsymbol{\theta}}_{D_{diss}} \end{aligned} \tag{16}$$

Consider the adaptive feedback control law

$$\begin{aligned} \boldsymbol{\tau} = {} & -A_e^{\mathrm{T}}K_{cp}\boldsymbol{\zeta}_e - (G - G_e)\bar{M}_AF\boldsymbol{\xi}_d + \\ & \bar{M}_AF\dot{\boldsymbol{\xi}}_d + \bar{D}_{diss}F\boldsymbol{\xi}_d - K_{cd}\boldsymbol{\xi}_e \end{aligned} \tag{17}$$

The parameters are estimated from the following parameters estimator

$$\dot{\bar{\boldsymbol{\theta}}} = -\Gamma Y_e^{\mathrm{T}}\boldsymbol{\xi}_e \tag{18}$$

where $K_{cp}$, $K_{cd}$, and $\Gamma$ are positive definite matrices, and

$$Y_e = [(G_e - G)Y(F\boldsymbol{\xi}_d) + Y(F\dot{\boldsymbol{\xi}}_d), Y(F\boldsymbol{\xi}_d)]_{3\times 6} \tag{19}$$

By substituting adaptive feedback control law (17) into the dynamic of error system (7), we can get the dynamic equation of closed-loop error system.

$$\begin{aligned} \dot{\boldsymbol{\xi}}_e = {} & M_A^{-1}GM_A(\boldsymbol{\xi}_e + F\boldsymbol{\xi}_d) - M_A^{-1}D_{diss}(\boldsymbol{\xi}_e + F\boldsymbol{\xi}_d) + \\ & G_e^{\mathrm{T}}F\boldsymbol{\xi}_d - M_A^{-1}A_e^{\mathrm{T}}K_{cp}\boldsymbol{\zeta}_e - M_A^{-1}(G - G_e)\bar{M}_AF\boldsymbol{\xi}_d - \\ & F\dot{\boldsymbol{\xi}}_d + M_A^{-1}\bar{M}_AF\dot{\boldsymbol{\xi}}_d + M_A^{-1}\bar{D}_{diss}F\boldsymbol{\xi}_d - M_A^{-1}K_{cd}\boldsymbol{\xi}_e = \\ & M_A^{-1}[(G_eM_A + M_AG_e^{\mathrm{T}})F\boldsymbol{\xi}_d + GM_A\boldsymbol{\xi}_e] + \\ & M_A^{-1}[(G_e - G)\tilde{M}_AF\boldsymbol{\xi}_d + \tilde{M}_AF\dot{\boldsymbol{\xi}}_d + \tilde{D}_{diss}F\boldsymbol{\xi}_d] - \\ & M_A^{-1}A_e^{\mathrm{T}}K_{cp}\boldsymbol{\zeta}_e - M_A^{-1}(D_{diss} + K_{cd})\boldsymbol{\xi}_e \end{aligned}$$

According to (15) and (16), there is

$$\begin{aligned} & (G_e - G)\tilde{M}_AF\boldsymbol{\xi}_d + \tilde{M}_AF\dot{\boldsymbol{\xi}}_d + \tilde{D}_{diss}F\boldsymbol{\xi}_d = \\ & (G_e - G)Y(F\boldsymbol{\xi}_d)\tilde{\boldsymbol{\theta}}_{M_A} + Y(F\dot{\boldsymbol{\xi}}_d)\tilde{\boldsymbol{\theta}}_{M_A} + Y(F\boldsymbol{\xi}_d)\tilde{\boldsymbol{\theta}}_{D_{diss}} = \\ & [(G_e - G)Y(F\boldsymbol{\xi}_d) + Y(F\dot{\boldsymbol{\xi}}_d),\ Y(F\boldsymbol{\xi}_d)]\tilde{\boldsymbol{\theta}} = Y_e\tilde{\boldsymbol{\theta}} \end{aligned}$$

So the dynamic equation of closed-loop error system is

$$\dot{\boldsymbol{\xi}}_e = M_A^{-1}[GM_A\boldsymbol{\xi}_e + (G_eM_A + M_AG_e^{\rm T})F\boldsymbol{\xi}_d] - M_A^{-1}A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e - M_A^{-1}(D_{diss}+K_{cd})\boldsymbol{\xi}_e + M_A^{-1}Y_e\tilde{\boldsymbol{\theta}} \quad (20)$$

**Theorem 1.** If the desired velocity $\boldsymbol{\xi}_d \neq 0$ and bounded, then, under the control of (17) and (18), the closed-loop error system (6) and (20) is locally uniformly asymptotically stable around the origin $(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = (\mathbf{0}, \mathbf{0})$.

**Proof.** Construct Lyapunov function as

$$V(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = \frac{1}{2}\boldsymbol{\zeta}_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e + \frac{1}{2}\boldsymbol{\xi}_e^{\rm T}M_A\boldsymbol{\xi}_e + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{\rm T}\Gamma^{-1}\tilde{\boldsymbol{\theta}} \quad (21)$$

Differentiating $V(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)$ with respect to the time along the trajectory of closed-loop error system (6) and (20), we have

$$\dot{V}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = \boldsymbol{\zeta}_e^{\rm T}K_{cp}\dot{\boldsymbol{\zeta}}_e + \boldsymbol{\xi}_e^{\rm T}M_A\dot{\boldsymbol{\xi}}_e + \tilde{\boldsymbol{\theta}}^{\rm T}\Gamma^{-1}\dot{\tilde{\boldsymbol{\theta}}} \quad (22)$$

Assume $\boldsymbol{\theta}$ is a constant. Since $\tilde{\boldsymbol{\theta}} = \bar{\boldsymbol{\theta}} - \boldsymbol{\theta}$, $\dot{\tilde{\boldsymbol{\theta}}} = \dot{\bar{\boldsymbol{\theta}}} = -\Gamma Y_e^{\rm T}\boldsymbol{\xi}_e$, and according to (20), we have

$$\begin{aligned}\dot{V}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = {}& \boldsymbol{\xi}_e^{\rm T}\{[GM_A\boldsymbol{\xi}_e + (G_eM_A + M_AG_e^{\rm T})F\boldsymbol{\xi}_d] - \\ & A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e - (D_{diss} + K_{cd})\boldsymbol{\xi}_e + Y_e\tilde{\boldsymbol{\theta}}\} - \\ & \tilde{\boldsymbol{\theta}}^{\rm T}Y_e^{\rm T}\boldsymbol{\xi}_e + \boldsymbol{\zeta}_e^{\rm T}K_{cp}\dot{\boldsymbol{\zeta}}_e = \\ & \boldsymbol{\xi}_e^{\rm T}[GM_A\boldsymbol{\xi}_e + (G_eM_A + M_AG_e^{\rm T})F\boldsymbol{\xi}_d] - \\ & \boldsymbol{\xi}_e^{\rm T}(D_{diss} + K_{cd})\boldsymbol{\xi}_e\end{aligned}$$

According to (8) ~ (10), we have

$$\begin{aligned}& GM_A\boldsymbol{\xi}_e + \left(G_eM_A + M_AG_e^{\rm T}\right)F\boldsymbol{\xi}_d = \\ & GM_A\boldsymbol{\xi}_e + \left(G_eM_A + M_AG_e^{\rm T}\right)(\boldsymbol{\xi} - \boldsymbol{\xi}_e) = \\ & GM_A\boldsymbol{\xi}_e + G_eM_A\boldsymbol{\xi} - G_eM_A\boldsymbol{\xi}_e - M_AG^{\rm T}\boldsymbol{\xi}_e = \\ & \begin{bmatrix} 0 & -m_{22}v & m_{11}u \\ m_{22}v & 0 & 0 \\ -m_{11}u & 0 & 0 \end{bmatrix}\boldsymbol{\xi}_e + \\ & (M_AG_e^{\rm T} - G_eM_A)\boldsymbol{\xi}_e + (GM_A - M_AG^{\rm T})\boldsymbol{\xi}_e\end{aligned}$$

Since the term in bracket is skew-symmetric, we have

$$\dot{V}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = -\boldsymbol{\xi}_e^{\rm T}(D_{diss} + K_{cd})\boldsymbol{\xi}_e \leq 0 \quad (23)$$

Applying Lyapunov uniform stability theorem[10], uniform stability is proved.

Now we prove the asymptotical stability. Consider the following assistant function

$$W(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = -\boldsymbol{\zeta}_e^{\rm T}K_{cp}A_e^{\rm T}M_A\boldsymbol{\xi}_e + \tilde{\boldsymbol{\theta}}^{\rm T}Y_e^{\rm T}M_A\boldsymbol{\xi}_e \quad (24)$$

Differentiate both sides of (24), then, we get

$$\begin{aligned}\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e) = {}& -\dot{\boldsymbol{\zeta}}_e^{\rm T}K_{cp}A_e^{\rm T}M_A\boldsymbol{\xi}_e - \boldsymbol{\zeta}_e^{\rm T}K_{cp}\dot{A}_e^{\rm T}M_A\boldsymbol{\xi}_e + \\ & \dot{\tilde{\boldsymbol{\theta}}}^{\rm T}Y_e^{\rm T}M_A\boldsymbol{\xi}_e + \tilde{\boldsymbol{\theta}}^{\rm T}\dot{Y}_e^{\rm T}M_A\boldsymbol{\xi}_e + (-\boldsymbol{\zeta}_e^{\rm T}K_{cp}A_e^{\rm T} + \tilde{\boldsymbol{\theta}}^{\rm T}Y_e^{\rm T})M_A\dot{\boldsymbol{\xi}}_e\end{aligned}$$

Substituting (20) into the above equation and letting $\boldsymbol{\xi}_e = 0$, i.e., $r_e = 0$, $u_e = 0$, and $v_e = 0$, which results in $G_e = 0$, and we have $M_A\dot{\boldsymbol{\xi}}_e = -A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e + Y_e\tilde{\boldsymbol{\theta}}$, and then,

$$\begin{aligned}& \dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} = \\ & (-\boldsymbol{\zeta}_e^{\rm T}K_{cp}A_e + \tilde{\boldsymbol{\theta}}^{\rm T}Y_e^{\rm T})(-A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e + Y_e\tilde{\boldsymbol{\theta}}) = \\ & (Y_e\tilde{\boldsymbol{\theta}} - A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e)^{\rm T}A_e^{\rm T}A_e(Y_e\tilde{\boldsymbol{\theta}} - A_e^{\rm T}K_{cp}\boldsymbol{\zeta}_e) = \\ & (A_eY_e\tilde{\boldsymbol{\theta}} - K_{cp}\boldsymbol{\zeta}_e)^{\rm T}(A_eY_e\tilde{\boldsymbol{\theta}} - K_{cp}\boldsymbol{\zeta}_e) \geq 0 \end{aligned} \quad (25)$$

According to (18), $\tilde{\boldsymbol{\theta}}$ is constant if $\boldsymbol{\xi}_e = 0$, and it denotes $\tilde{\boldsymbol{\theta}}_0$.

If $\tilde{\boldsymbol{\theta}}_0 = 0$, then, according to (25), $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} = \boldsymbol{\zeta}_e^{\rm T}K_{cp}^{\rm T}K_{cp}\boldsymbol{\zeta}_e$. Then, we know that $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} = 0$ if and only if $\boldsymbol{\zeta}_e = 0$. So, we can conclude that $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} > 0$ if $0 < \|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\| < \delta$ ( any $\delta > 0$).

If $\tilde{\boldsymbol{\theta}}_0 \neq 0$, then, according to (25), $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} = 0$ if and only if

$$\boldsymbol{\zeta}_e = K_{cp}^{-1}A_eY_e\tilde{\boldsymbol{\theta}}_0 \quad (26)$$

We can get $\boldsymbol{\zeta}_e = \boldsymbol{\zeta}_e^0$ by solving nonlinear algebra equation (26), and we denote $\|\boldsymbol{\zeta}_e^0\| = \gamma$. Then, $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} > 0$ if $0 < \|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\| < \gamma$. We can also find out a strictly increasing function $\delta(\|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\|)|_{\boldsymbol{\xi}_e=0}$, so that $\dot{W}(\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e)|_{\boldsymbol{\xi}_e=0} \geq \delta(\|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\|)|_{\boldsymbol{\xi}_e=0}$ at the region $0 < \|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\| < \gamma$. This satisfies the condition (5) of the Matrosv theorem[11]. From above description, conditions (1) ~ (4) of Matrosov theorem are satisfied by (6), (20) ~ (24) when we select $V^*(\boldsymbol{x}) = -\boldsymbol{\xi}_e^{\rm T}D_{diss}\boldsymbol{\xi}_e$. Hence, the all conditions of Matrosov theorem are satisfied at the region $0 < \|\boldsymbol{\zeta}_e, \boldsymbol{\xi}_e\| < \gamma$. Then, the local asymptotic stability of error system is guaranteed. □

From the control law, we can find that if $\boldsymbol{\xi}_d = 0$, i.e., keeping the airship hovering at the desired point (this is an important task for airship used as a communication platform), the parameter estimator (18) is trivial, and the adaptive control law (17) reduces to the nonadaptive version.

## 3 Simulation

The effectiveness of the control law is illustrated by the following simulation. The parameters are selected as $J_{33} = 12\,167.3\,\mathrm{Kg}\cdot\mathrm{m}^2$, $m_{11} = 301.9\,\mathrm{Kg}$, $m_{22} = 455.1\,\mathrm{Kg}$, $d_r = 73\,\mathrm{Kg/s}$, $d_u = 50\,\mathrm{Kg/s}$, and $d_v = 50\,\mathrm{Kg/s}$. In the period from 20 s to 30 s, there are disturbances with 15 m/s in $u$ direction, and white noise $|\omega| \leq 3$ in $v$ and $r$ directions. The desired velocity is $\boldsymbol{\xi}_d = [\ 0.1\sin(0.1\times t) \quad 10 \quad 0\ ]^{\rm T}$. The control parameters are taken as $K_{cp} = \mathrm{diag}\{100\,000, 1\,000, 1\,000\}$, $K_{cd} = \mathrm{diag}\{60\,000, 300, 300\}$, and $\Gamma = \mathrm{diag}\{6, 0.3, 0.3, 0.3, 0.3, 0.3\}$. The initial value of parameter estimator is 80 % of real parameter value. The results are shown in Figs. 2 ~ 6.

The desired trajectory is a dashed sine wave as shown in Fig. 5. At the beginning, the real trajectory of the airship fast verges to the desired one. It deviates from the desired trajectory at about 20 s since wind disturbance occurs but the deviation is in some range with the effect of controller. From Fig. 4, we can see that the control inputs vary slowly when disturbance occurs at $t = 0$ and varying fast when disturbance vanishes. That is because the controller consists of system error feedback but without disturbance feed-forward. The controller outputs vary with the system errors, which vary slowly from zero at the beginning of the disturbance. When disturbance vanishes at $t = 30$ s, the system errors are still great so that the control action will be strong, which makes the system errors converge to zero with some oscillation as shown in Figs. 2 and 3. Fig. 6 shows that the norm of parameter error $\tilde{\boldsymbol{\theta}}$. The results verify the performance of the proposed controller.

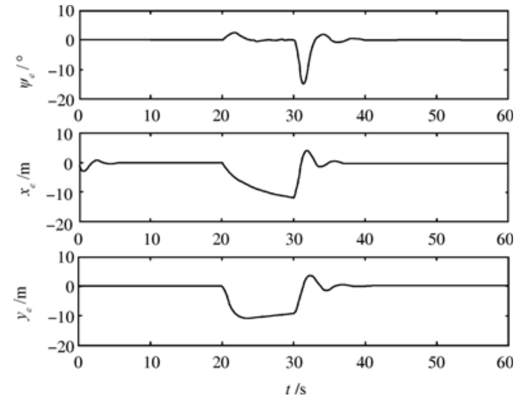


Fig. 2 Configuration errors

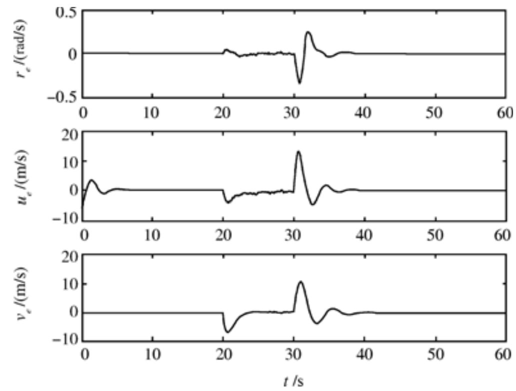


Fig. 3 Velocity errors

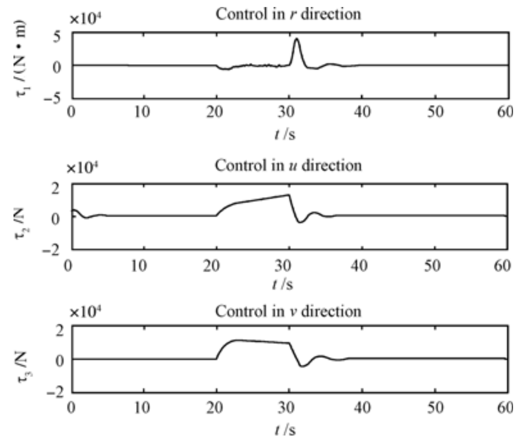


Fig. 4 Control inputs

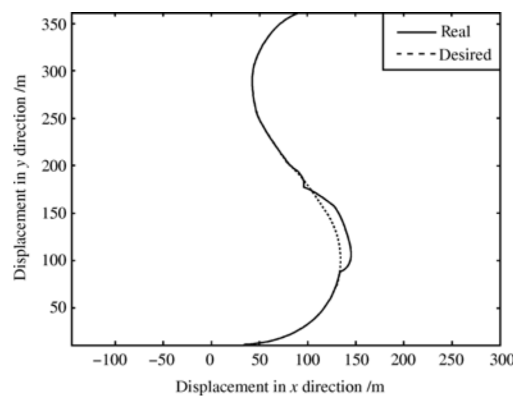


Fig. 5 Real and desired trajectories

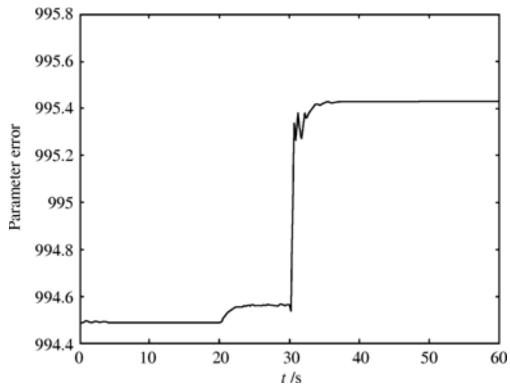


Fig. 6 Norm of parameter estimator error

## 4 Conclusion

In this paper, we presented a new approach to design a robust adaptive controller using Lyapunov stability method and Matrosov theorem for an airship with neutral buoyancy. By introducing a new definition of airship′s configuration and velocity errors, we established the error kinematics and dynamics systems. Hence, the configuration stabilization problem and the trajectory tracking problem could be transformed into an unified framework, i.e., the stabilization problem of error system. Especially, the controller needs no knowledge of the parameter uncertainty in the case of set-point control. The simulation results verified the performance of the controller.

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