An Integrated Navigation System of NGIMU/ GPS Using a Fuzzy Logic Adaptive Kalman Filter

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Abstract. The Non-gyro inertial measurement unit (NGIMU) uses only accelerometers replacing gyroscopes to compute the motion of a moving body. In a NGIMU system, an inevitable accumulation error of navigation parameters is produced due to the existence of the dynamic noise of the accelerometer output. When designing an integrated navigation system, which is based on a proposed nine-configuration NGIMU and a single antenna Global Positioning System (GPS) by using the conventional Kalman filter (CKF), the filtering results are divergent because of the complicity of the system measurement noise. So a fuzzy logic adaptive Kalman filter (FLAKF) is applied in the design of NGIMU/GPS. The FLAKF optimizes the CKF by detecting the bias in the measurement and prevents the divergence of the CKF. A simulation case for estimating the position and the velocity is investigated by this approach. Results verify the feasibility of the FLAKF.

1 Introduction

Most current inertial measurement units (IMU) use linear accelerometers and gyroscopes to sense the linear acceleration and angular rate of a moving body respectively. In a non-gyro inertial measurement unit (NGIMU) [1-6], accelerometers are not only used to acquire the linear acceleration, but also replace gyroscopes to compute the angular rate according to their positions in three-dimension space. NGIMU has the advantages of anti-high g value shock, low power consumption, small volume and low cost. It can be applied to some specific occasions such as tactic missiles, intelligent bombs and so on.

But due to the existence of the dynamic noise of the accelerometer output, it is inevitable that the system error increases quickly with time by integrating the accelerometer output. The best method to solve this problem above is the application of the integrated navigation system. NGIMU/GPS integrated navigation system can fully exert its superiority and overcome its shortcomings to realize the real-time high precision positioning in a high kinematic and strong electrically disturbed circumstance. But when using the conventional Kalman filter (CKF) in the NGIMU/GPS, the filtering results are often divergent due to the uncertainty of the statistical characteristics of dynamic noise of the accelerometer output and the system measurement noise. So, in order to ascertain the statistical characteristics of the noises mentioned above and

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alleviate the consumption error, a new fuzzy logic adaptive Kalman filter (FLAKF) [7] is proposed in designing a NGIMU/GPS integrated navigation system.

2 Accelerometer Output Equation

As all know, the precession of gyroscopes can be used to measure the angular rate. Based on this principle, IMU measures the angular rate of a moving body. The angle value can be obtained by integrating the angular rate with given initial conditions. With this angle value and the linear acceleration values in three directions, the current posture of the moving body can be estimated.

The angular rate in a certain direction can be calculated by using the linear acceleration between two points. To obtain the linear and angular motion parameters of a moving body in three-dimension space, the accelerometers need to be appropriately distributed on the moving body and the analysis of the accelerometer outputs is needed.

An inertial frame and a rotating moving body frame are exhibited in Fig. 1, where b represents the moving body frame and I the inertial frame.

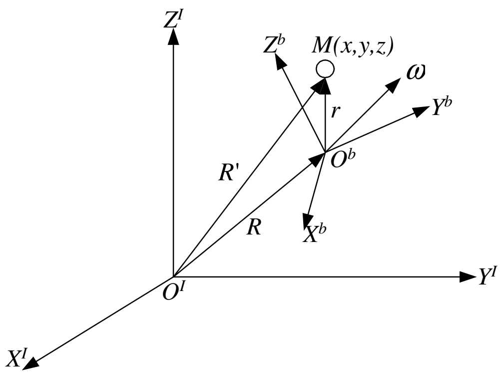

Fig. 1. Geometry of body frame (b) and inertial frame (I)

The acceleration of point M is given by

$$\boldsymbol{a} = \ddot{\boldsymbol{R}}_I + \ddot{\boldsymbol{r}}_b + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_b + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) , \tag{1}$$

where $\ddot{\boldsymbol{r}}_b$ is the acceleration of point M relative to body frame. $\ddot{\boldsymbol{R}}_I$ is the inertial acceleration of O^b relative to O^I . $2\boldsymbol{\omega} \times \dot{\boldsymbol{r}}_b$ is known as the Coriolis acceleration, $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ represents a centripetal acceleration, and $\dot{\boldsymbol{\omega}} \times \boldsymbol{r}$ is the tangential acceleration owing to angular acceleration of the rotating frame

If M is fixed in the b frame, the terms $\dot{\boldsymbol{r}}_b$ and $\ddot{\boldsymbol{r}}_b$ vanish. And Eq.(1) can be rewritten as

$$\boldsymbol{a} = \ddot{\boldsymbol{R}}_I + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) . \tag{2}$$

Thus the accelerometers rigidly mounted at location $\boldsymbol{r}_i$ on the body with sensing direction $\boldsymbol{\theta}_i$ produce $\boldsymbol{A}_i$ as outputs.

$$\boldsymbol{A}_i = [\ddot{\boldsymbol{R}}_I + \dot{\boldsymbol{\Omega}}\boldsymbol{r}_i + \boldsymbol{\Omega\Omega r}_i]\cdot\boldsymbol{\theta}_i \qquad (i = 1,2,\ldots,N)\,, \tag{3}$$

where

$$\boldsymbol{\Omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix},\ \ddot{\boldsymbol{R}}_I = \begin{bmatrix} \ddot{R}_{Ix} \\ \ddot{R}_{Iy} \\ \ddot{R}_{Iz} \end{bmatrix}. \tag{4}$$

3 Nine-Accelerometer Configuration

In this research, a new nine-accelerometer configuration of NGIMU is proposed. The locations and the sensing directions of the nine accelerometers in the body frame are shown in Fig.2. Each arrow in Fig.2 points to the sensing direction of each accelerometer.

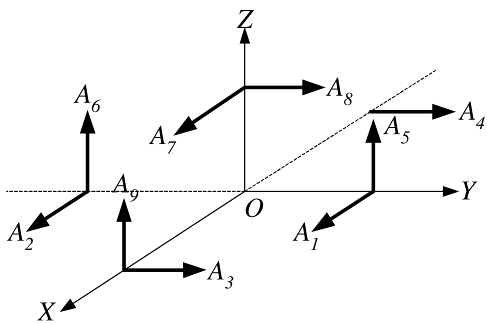

Fig. 2. Nine-accelerometer configuration of NGIMU

The locations and sensing directions of the nine accelerometers are

$$[\boldsymbol{r}_1,\cdots,\boldsymbol{r}_9] = l\begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}, \tag{5}$$

where l is the distance between the accelerometer and the origin of the body frame.

$$[\boldsymbol{\theta}_1,\cdots,\boldsymbol{\theta}_9] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

It is easy to obtain

$$[\boldsymbol{r}_1 \times \boldsymbol{\theta}_1, \cdots, \boldsymbol{r}_9 \times \boldsymbol{\theta}_9] = l \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

With Eq.(3), we get the accelerometer output equation

$$\boldsymbol{A}_i = \begin{bmatrix} 0 & 0 & -l & 1 & 0 & 0 \\ 0 & 0 & l & 1 & 0 & 0 \\ 0 & 0 & l & 0 & 1 & 0 \\ 0 & 0 & -l & 0 & 1 & 0 \\ l & 0 & 0 & 0 & 0 & 1 \\ -l & 0 & 0 & 0 & 0 & 1 \\ 0 & l & 0 & 1 & 0 & 0 \\ -l & 0 & 0 & 0 & 1 & 0 \\ 0 & -l & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \\ \ddot{R}_{Ix} \\ \ddot{R}_{Iy} \\ \ddot{R}_{Iz} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & l \\ 0 & 0 & 0 & 0 & 0 & -l \\ 0 & 0 & 0 & 0 & 0 & l \\ 0 & 0 & 0 & 0 & 0 & -l \\ 0 & 0 & 0 & l & 0 & 0 \\ 0 & 0 & 0 & -l & 0 & 0 \\ 0 & 0 & 0 & 0 & l & 0 \\ 0 & 0 & 0 & l & 0 & 0 \\ 0 & 0 & 0 & 0 & l & 0 \end{bmatrix} \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \\ \omega_y \omega_z \\ \omega_x \omega_z \\ \omega_x \omega_y \end{bmatrix}. \tag{8}$$

With Eq.(8), the linear expressions are

$$\dot{\omega}_x = \frac{1}{4l}(A_3 + A_4 + A_5 - A_6 - 2A_8), \tag{9a}$$

$$\dot{\omega}_y = \frac{1}{4l}(-A_1 - A_2 + A_5 + A_6 + 2A_7 - 2A_9), \tag{9b}$$

$$\dot{\omega}_z = \frac{1}{4l}(-A_1 + A_2 + A_3 - A_4), \tag{9c}$$

$$\ddot{R}_{Ix} = \frac{1}{2}(A_1 + A_2), \; \ddot{R}_{Iy} = \frac{1}{2}(A_3 + A_4), \; \ddot{R}_{Iz} = \frac{1}{2}(A_5 + A_6). \tag{9d}$$

4 Conventional Kalman Filter (CKF)

In Eq.(9), the linear acceleration and the angular acceleration are all expressed as the linear combinations of the accelerometer outputs. The conventional algorithm computes the navigation parameters as the time integration or double integrations of the equations in Eq.(9). But a numerical solution for the navigation parameters depends on the value calculated from previous time steps. And if the accelerometer output has a dynamic error, the error of the navigation parameters will inevitably increase with t and t^2 rapidly. So the design of a NGMIMU/GPS integrated navigation system is expected. In this section, the CKF is used in the system.

In order to analyze the problem in focus, we ignore the disturbance error contributed to the accelerometers due to the difference of the accelerometers' sensing directions in three-dimension space. Define the states vector $\boldsymbol{X}(t)$ for the motion as

$$\boldsymbol{X}(t) = \left[S_e(t) \quad S_N(t) \quad V_e(t) \quad V_N(t) \quad \omega(t)\right]^T, \tag{10}$$

where $S_e(t)$ is the estimation eastern position of the moving body at time t with respect to the earth frame (as inertial frame), $S_N(t)$ is the estimation northern position, $V_e(t)$ is the estimation eastern velocity, $V_N(t)$ is the estimation northern velocity and $\omega(t)$ is the estimation angular rate along x axis. Considering the relationship between the parameters, the states equations are then

$$\dot{S}_e = V_e,\ \dot{S}_N = V_N,\ \dot{V}_e = a_e,\ \dot{V}_N = a_N,\ \dot{\omega} = \dot{\omega}_x, \tag{11}$$

where

$$a_e = T_{11}\ddot{R}_{Ix} + T_{21}\ddot{R}_{Iy} + T_{31}\ddot{R}_{Iz},\ a_N = T_{12}\ddot{R}_{Ix} + T_{22}\ddot{R}_{Iy} + T_{32}\ddot{R}_{Iz}. \tag{12}$$

In Eq.(12), T_{11}, T_{21}, T_{31}, T_{12}, T_{22} and T_{32} are the components of the coordinate transform matrix $\boldsymbol{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$.

We also obtain

$$\dot{\omega}_x = \frac{1}{4l}(A_3 + A_4 + A_5 - A_6 - 2A_8). \tag{13}$$

The system state equation and system measurement equation in matrix form become

$$\dot{\boldsymbol{X}} = \boldsymbol{\Psi X} + \boldsymbol{Gu} + \boldsymbol{\Gamma W}, \tag{14}$$

and

$$\boldsymbol{Z} = \boldsymbol{HX} + \boldsymbol{\varepsilon}. \tag{15}$$

In the system measurement equation Eq.(15), the input vector $\boldsymbol{X}$ is the output of the GPS receiver (position and velocity). In Eq.(14) and Eq.(15), $\boldsymbol{\varepsilon}$ and $\boldsymbol{W}$ denote the measurement noise matrix and the dynamic noise matrix respectively.

The preceding results are expressed in continuous form. Equation of state and measurement for discrete time may be deduced by assigning $t = kT$, where $k = 1,2,...$, and T denotes the sampling period. Straightforward application of the discrete time Kalman filter to (14) and (15) yields the CKF algorithm as outlined below. $\hat{\boldsymbol{X}}_{0/0}$ is the initial estimate of the CKF state vector. $\boldsymbol{P}_{0/0}$ is the initial estimate of the CKF state vector error covariance matrix.

5 Fuzzy Logic Adaptive Kalman Filter (FLAKF)

The CKF algorithm mentioned in the above section requires that the dynamic noise and the system measurement noise process are exactly known, and the noises processes are zero mean white noise. In practice, the statistical characteristics of the noises are uncertain. In the GPS measurement equation Eq.(15), among the measurement noise ε, the remnant ionosphere delay modified by the ionosphere model is just not zero mean white noise. Furthermore, the value of the dynamic error of the accelerometer output also cannot be exactly obtained in a kinematic NGIMU/GPS positioning. These problems result in the calculating error of $\boldsymbol{K}_k$ and make the filtering process divergent. In order to solve the divergence due to modeling error, In this paper, a fuzzy logic adaptive Kalman filter (FLAKF) is proposed to adjust the exponential weighting of a weighted CKF and prevent the Kalman filter from divergence. The fuzzy logic adaptive Kalman filter will continually adjust the noise strengths in the filter's internal model, and tune the filter as well as possible. The structure of NGIMU/GPS using FLAKF is shown in Fig. 3.

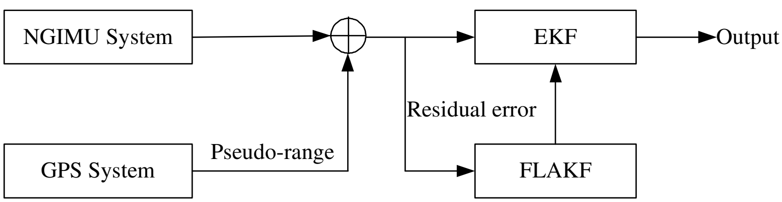

Fig. 3. Structure of NGIMU/GPS using FLAKF

The fuzzy logic is a knowledge-based system operating on linguistic variables. The advantages of fuzzy logic with respect to more traditional adaptation schemes are the simplicity of the approach and the application of knowledge about the controlled system. In this paper, FLAKF is to detect the bias in the measurement and prevent divergence of the CKF. Let us assume the model covariance matrices as

$$\begin{cases} \boldsymbol{R}_k = \alpha \boldsymbol{R}_v \\ \boldsymbol{Q}_k = \beta \boldsymbol{Q}_v \end{cases}, \tag{16}$$

where α and β are the adjustment ratios which are time-varying. The value of α and β can be acquired from the outputs of FLAKF. Let us define $\boldsymbol{\delta} = \boldsymbol{Z}_i - \boldsymbol{H}\hat{\boldsymbol{X}}_{k/k-1}$ as residual error, which reflects the degree to which the NGIMU/GPS model fits the data. According to the characteristic of the residual error of CKF, the variance matrixes of the dynamic noise of the accelerometer output and the measurement noise can be adjust self-adaptively using α and β. If $\alpha = \beta = 1$, we obtain a regular CKF.

The good way to verify whether the Kalman filter is performing as designed is to monitor the residual error. It can be used to adapt the filter. In fact, the residual error $\boldsymbol{\delta}$ is the difference between the actual observing results and the measurement predictions based on the filter model. If a filter is performing optimally, the residual error is a zero-mean white noise process. The covariance of residual error $\boldsymbol{P}_r$ relates to $\boldsymbol{Q}_v$ and $\boldsymbol{R}_v$. The covariance of the residual error is given by:

$$\boldsymbol{P}_r = \boldsymbol{H}(\boldsymbol{\Psi}\boldsymbol{P}_{k-1}\boldsymbol{\Psi}^T + \boldsymbol{Q}_v)\boldsymbol{H}^T + \boldsymbol{R}_v . \quad (17)$$

Using some traditional fuzzy logic system for reference, the Takagi-Sugeno fuzzy logic system is used to detect the divergence of CKF and adapt the filter. According to the variance and the mean of the residual error, two fuzzy rule groups are built up. To improve the performance of the filter, the two groups calculate the proper α and β respectively, and readjust the covariance matrix $\boldsymbol{P}_r$ of the filter.

As an input to FLAKF, the covariance of the residual error and the mean value of the residual error are used in order to detect the degree of the divergence. By choosing n to provide statistical smoothing, the mean and the covariance of the residual error are

$$\overline{\boldsymbol{\delta}} = \frac{1}{n}\sum_{j=t-n}^{t}\boldsymbol{\delta}_j , \quad (18)$$

$$\overline{\boldsymbol{P}}_r = \frac{1}{n}\sum_{j=t-n+1}^{t}\boldsymbol{\delta}_j\boldsymbol{\delta}_j^T . \quad (19)$$

The estimated value $\overline{\boldsymbol{P}}_r$ can be compared with its theoretical value $\boldsymbol{P}_r$ calculated from CKF. Generally, when covariance $\overline{\boldsymbol{P}}_r$ is becoming larger than theoretical value $\boldsymbol{P}_r$, and mean value $\overline{\boldsymbol{\delta}}$ is moving from away zero, the Kalman filter is becoming unstable. In this case, a large value of β is applied. A large β means that process noises are added and we are giving more credibility to the recent data by decreasing the noise covariance. This ensures that all states in the model are sufficiently excited by the process noise. Generally, $\boldsymbol{R}_v$ has more impact on the covariance of the residual error. When the covariance is extremely large and the mean takes the values largely different from zero, there are presumably problems with GPS measurements. Therefore, the filter cannot depend on these measurements anymore, and a smaller α will be used. By selecting the appropriate α and β , the fuzzy logic controller optimally adapt the Kalman filter and tries to keep the innovation sequence act as a zero-mean white noise. The membership functions of the covariance and the mean value of the residual error are also built up.

6 Simulations and Results

The simulations of NGIMU/GPS using the CKF and the FLAKF are performed respectively in this section. Fig. 4, Fig. 5, Fig. 6 and Fig .7 illustrate the eastern position estimating error, the northern position estimating error, the eastern velocity estimating error and the northern velocity estimating error respectively.

In this simulation, the GPS receiver used is the Jupiter of Rockwell Co.. The initial conditions in position, velocity, posture angle and angular rate are $x(0)=0$ m, $y(0)=0$ m, $z(0)=0$ m, $v_x(0)=0$ m/s, $v_y(0)=0$ m/s, $v_z(0)=0$ m/s, $\alpha_x=0$ rad , $\alpha_y=0$ rad ,

$\alpha_z = \pi/3$ rad, $\omega_x(0) = 0$ rad/s, $\omega_y(0) = 0$ rad/s, $\omega_z(0) = 0$ rad/s respectively. The accelerometer static bias is 10^{-5}g and the swing of posture angle is 0.2 rad. Moreover, when using the CKF, assume that $\boldsymbol{W}$ and $\boldsymbol{\varepsilon}$ are all Gaussian distribution, the covariance are $\boldsymbol{Q}_v = (0.01)\boldsymbol{I}_{9\times9}$ and $\boldsymbol{R}_v = (0.01)\boldsymbol{I}_{5\times5}$ respectively, and $\boldsymbol{P}_{0/0} = (0.01)\boldsymbol{I}_{5\times5}$. The time required for simulation is 100s, and that for sampling is 10ms.

Comparing the curves in Fig 4 and Fig. 5, it is obvious that the eastern position estimating error and the northern position estimating error of NGIMU/GPS using the two filtering approaches are all leveled off after estimating for some time. And the errors acquired with the FLAKF are less than those with the CKF. In Fig.4, the error drops from 160m to 50m after using the FLAKF at 100s. In Fig.5, that is 220m to 100m. The similar results are also acquired in Fig.6 and Fig. 7 in the velocity estimation. The curves indicate that the NGIMU/GPS with the FLAKF can effectively alleviate the error accumulation of the estimation of the navigation parameters. When a designer lacks sufficient information to develop complete models or the parameters will slowly change with time, the FLAKF can be used to adjust the performance of CKF on-line.

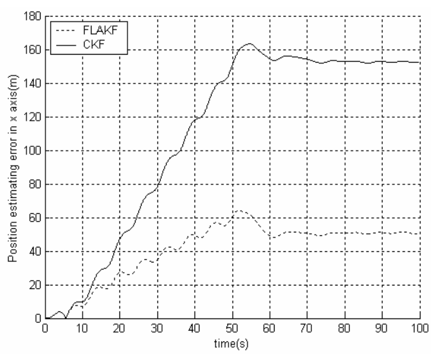

Fig. 4. The estimating error of the eastern position

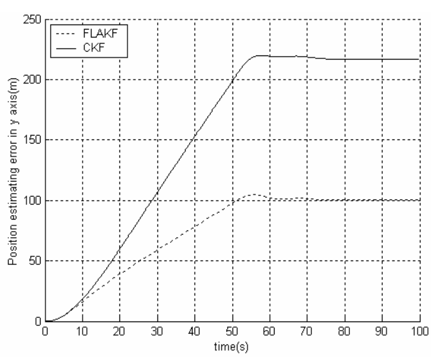

Fig. 5. The estimating error of the northern position

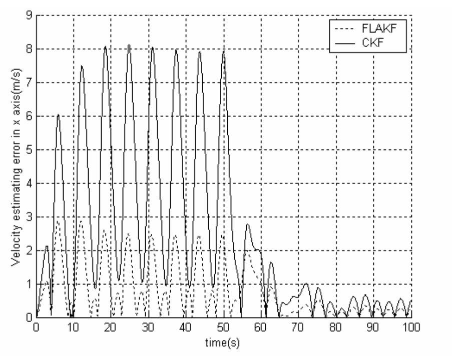

Fig. 6. The estimating error of the eastern velocity

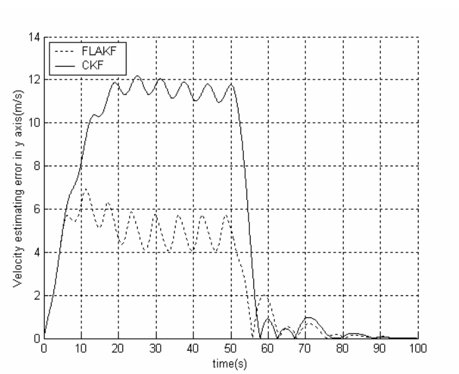

Fig. 7. The estimating error of the northern velocity

6 Conclusions

Due to the existence of the dynamic noise of the accelerometer output, it is inevitable that the navigation parameter estimation error increases quickly with time by integrating the accelerometer output. The use of the FLAKF to design a NGIMU/GPS based on a NGIMU of nine-accelerometer configuration can overcome the uncertainty of the statistical characteristics of the noises and alleviate the errors accumulation speed. By monitoring the innovations sequence, the FLAKF can evaluate the performance of a CKF. If the filter does not perform well, it would apply two appropriate weighting factors α and β to improve the accuracy of a CKF.

In FLAKF, there are 9 rules and therefore, little computational time is needed. It can be used to navigate and guide autonomous vehicles or robots and achieved a relatively accurate performance. Also, the FLAKF can use lower state-model without compromising accuracy significantly. Another words, for any given accuracy, the FLAKF may be also to use a lower order state model.

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