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# Accounting for Elastic Energy Storage in McKibben Artificial Muscle Actuators

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## ABSTRACT

The McKibben artificial muscle is a pneumatic actuator whose properties include a very high force to weight ratio. This characteristic makes it very attractive for a wide range of applications such as mobile robots and prosthetic appliances for the disabled. In this paper we present a model that includes a non-linear, Mooney-Rivlin mathematical description of the actuator's internal bladder. Experimental results show that the model provides improvement in the ability to predict the actuator's output force. However, a discrepancy between model and experiment, albeit smaller than previous models, still exists. A number of factors are identified that may be responsible for this discrepancy.

## 1 Introduction

The ability to operate in an uncontrolled environment remains one of the most difficult problems in robotic research. One approach to solving this problem involves emulating human and animal models as both regularly interact with the environment in a robust manner. In support of such an approach, we are currently developing a muscle-like actuator that includes a pneumatic device once used as an orthotic appliance for polio patients (Nickel et al., 1963). Powered by compressed gas, the McKibben actuator (see figure 1) is made from an inflatable inner bladder sheathed with a double helical braid which contracts lengthwise when expanded radially (Gaylord, 1958).

Our interest in a mathematical model of the McKibben actuator was to better understand the design parameters in order to improve desirable characteristics (e.g. output force v. input pressure) while minimizing undesirable characteristics (e.g. fatigue properties). Previous efforts to develop a theoretical model of the McKibben actuator have relied on braid characteristics alone and ignored bladder properties (e.g. Gaylord, 1958; Schulte, 1961; Tondu et al., 1994; Paynter, 1996). A few investigators have plotted their model predictions versus experimental results, but no theoretical model has achieved satisfactory results. This paper will present an improved model that includes the material properties of the inner bladder constrained by braid kinematics and will compare the model predictions with experimental results.

## 2 Theoretical Model

Using conservation of energy and assuming the actuator maintains $dV/dP$ equal to zero, reasonable for actuators built with stiff braid fibers that are always in contact with the inner bladder, the tensile force produced by this type of actuator can be calculated from:

$$F = P\frac{dV}{dL} - V_b\frac{dW}{dL} \tag{1}$$

where $P$ is the input actuation pressure, $dV$ is the change in the actuator's interior volume, $dL$ is the change in the actuator's length, $V_b$ is the volume occupied by the bladder, and $dW$ is the change in strain energy density (also known as the change in stored energy on a per volume basis). Neglecting the second term on the right hand side of equation (1) and assuming the actuator maintains the form of a right circular cylinder yields known solutions (Gaylord, 1958; Schulte, 1961; Chou and Hannaford, 1996). This solution, formulated in terms of the longitudinal stretch ratio ($\lambda_1$), is given by:

$$F_{\text{gaylord}} = \frac{P}{4N^2\pi}\left[3(\lambda_1 L_o)^2 - B^2\right] \tag{2}$$

where $F_{\text{gaylord}}$ is the actuator force, $N$ is the number of turns a single helical thread makes about the diameter of the actuator over it length, and $B$ is the length of that thread (see figure 1 and table 1). $\lambda_1$ is given by $\lambda_1 = L_i/L_o$, where $L_i$ is the actuator's instantaneous length and $L_o$ is the original, resting state length.

The approach to solving the second term on the right hand side of equation (1) for a rubber bladder of known geometry and material properties is based on a non-linear materials model developed by Mooney and Rivlin in the 1940's and 1950's (e.g. Treloar, 1958). Their well-known research proposed a relationship between stress ($\bar{\sigma}$) and strain ($\bar{\varepsilon}$) given by $\bar{\sigma} = dW/d\bar{\varepsilon}$ where $W$ is the strain energy density function. Using the assumptions of initial isotropsy and incompressibility, $W$ can be described as a function of two strain invariants ($I_1$ and $I_2$):

$$W = \sum_{i=0,j=0}^{\infty} C_{ij}(I_1 - 3)^i (I_2 - 3)^j \tag{3}$$

where $C_{ij}$ are empirical constants (Treloar, 1958). Published elsewhere is the specific relationship between the longitudinal and circumferential stretch ratios that arises from the braid kinematics (Klute and Hannaford, 1998). Only two Mooney-Rivlin constants ($C_{10}$ =118.4 kPa and $C_{01}$ =105.7 kPa) were necessary for accurate results with the natural latex rubber bladder.

Solving equation (1) using the non-linear Mooney-Rivlin materials model results in a McKibben actuator model whose rubber bladder is allowed to deform and store elastic energy in a non-linear fashion. This model is given by:

$$F_{mr} = P\left\{\frac{3(\lambda_1 L_o)^2 - B^2}{4N^2\pi}\right\} - V_b\left\{\begin{array}{l} \frac{1}{2L_o{}^3\lambda_1{}^3}\left\{4(C_{10}+C_{01})L_o{}^2\left(-1+\lambda_1{}^4\right)\right. \\ + \frac{4L_o{}^6(-1+\lambda_1)\lambda_1{}^2(1+\lambda_1)(C_{10}+C_{01}\lambda_1{}^2)}{\left[-4N^2\pi^2R_o{}^2+L_o{}^2\left(-1+\lambda_1{}^2\right)\right]^2} \\ - \frac{4L_o{}^4\left(C_{10}+C_{01}\lambda_1{}^4\right)}{-4N^2\pi^2R_o{}^2+L_o{}^2\left(-1+\lambda_1{}^2\right)} \\ \left. - \frac{L_o{}^4\lambda_1{}^4\left[C_{10}+C_{01}\left(-1+2\lambda_1{}^2\right)\right]}{N^2\pi^2R_o{}^2}\right\} \end{array}\right\} \quad (4)$$

where $F_{mr}$ is the predicted force, $R_o$ is the resting state radius, and bladder volume ($V_b$) is calculated using the radius, length, and bladder thickness ($t_o$) data given in table 1.

**3 Experimental Results**

To test the predictive value of the model, we conducted a series of experiments on several different sized McKibben actuators. We used an axial-torsional Bionix™ testing instrument (MTS Systems Corp., Minnesota, U.S.A.) to apply uniaxial displacements while measuring the output force at specific input pressures. To minimize tip-effects at the actuator's ends (a violation of the theoretical right circular cylinder assumption), the three different sized actuators (see table 1) were constructed such that their lengths to diameter ratios were at least 14.

The results from the largest size actuator comparing the experimental measurements with the model predictions are shown in figure 2. Gaylord's model ($F_{gaylord}$, equation 2), based strictly on braid geometry, over-predicted the actual force by approximately 500 N over the entire contraction range at 5 bar. The model incorporating bladder geometry and Mooney-Rivlin material properties ($F_{mr}$, equation 4) also over-predicted the actual force, but by only 275 N at 5 bar. Proportional results for both models were also obtained at activation pressures of 2, 3, and 4 bar, but are not shown. Also not shown are similar results were obtained for the smaller actuators.

**4 Discussion**

Accounting for the bladder geometry and its material properties yields a more accurate model and provides insights for the designer. For example, the effect of altering bladder thickness or selecting another bladder material (e.g. synthetic silicone rubber instead of natural latex rubber) can be examined using equation (4). However, a significant discrepancy remains between the model and the experimental results. Likely factors that may account for the difference includes other mechanisms of elastic energy storage and the effects of friction between the bladder and the braid, as well as between the fibers of the braid itself. These factors can be expected to be a function of (1) braid material, (2) bladder material, (3) activation pressure, and (4) instantaneous actuator length and are likely to be highly non-linear.

**5 Conclusion**

Advances in the theoretical modeling of the McKibben actuator have been presented with the objective of identifying the dependence of performance on the properties of both the exterior braided shell and the inner bladder. By modeling the inner bladder as an incompressible Mooney-Rivlin material of known geometry, we have been able to improve predictions of the actuator's output force. However, further work in understanding other mechanisms of energy storage and frictional effects is required.

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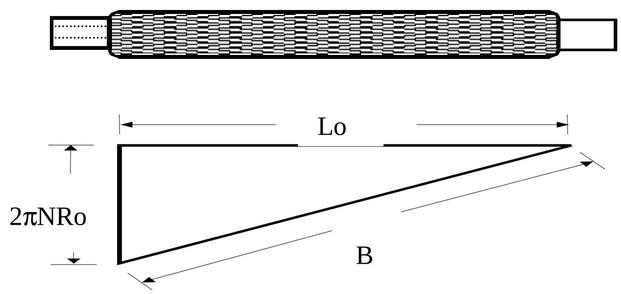
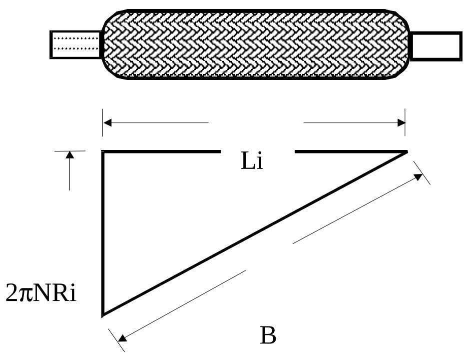



(1a)




(1b)

Figure 1: McKibben actuators are fabricated from two principle components: an inflatable inner bladder made of a rubber material and an exterior braided shell wound in a double helix. At ambient pressure, the actuator is at its resting length (figure 1a). As pressure increases, the actuator contracts proportionally until it reaches its maximally contracted state at maximum pressure (figure 1b). Both the thread length ($B$) and the number of turns an individual thread makes about the diameter ($N$) are constant. The amount of contraction is described by the actuator's longitudinal stretch ratio given by $\lambda_1 = L_i / L_o$ where $L$ is the actuator's length, and subscript $_i$ refers to the instantaneous dimension and the subscript $_o$ refers to the original, resting state dimension.

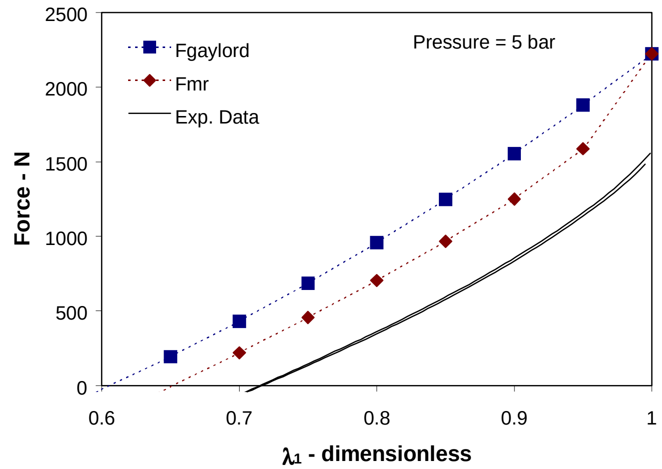



Figure 2: Model predictions versus experimental results are presented for the largest of the three actuators tested (nominal braid diameter of 1-1/4). $F_{\text{gaylord}}$ refers to the model published by Gaylord (1958) which does not account for bladder geometry or material. $F_{\text{mr}}$ refers to our model which incorporates both bladder geometry and Mooney-Rivlin material properties.

Table 1: Model constants for three different sized McKibben actuators. The braid for each actuator was constructed from a polyester thread and the bladder of each was made of natural latex rubber.

| General Dimensions | | | |
|---|---|---|---|
| Braid[1] | 1-1/4 | 3/4 | 1/2 |
| Bladder[2] | 1/2 x 3/32 | 3/8 x 1/16 | 5/32 x 3/64 |
| Braid Dimensions | | | |
| $N$ - turns | 1.5 | 1.5 | 1.7 |
| $L_o$ - mm | 264.0 | 181.0 | 126.0 |
| $B$ - mm | 277.1 | 190.6 | 130.5 |
| Bladder Dimensions | | | |
| $R_o$ - mm | 8.7 | 6.4 | 3.2 |
| $t_o$ - mm | 2.4 | 1.6 | 1.2 |

[1]The braid is described by the manufacturer's designation for nominal diameter in inches (i.e. 1-1/4 = 1.25 in). Alpha Wire Corp., Elizabeth, New Jersey, U.S.A.
[2]The bladder is described by the internal diameter (inches) and the wall thickness (inches) following the convention typical of most manufacturers.